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House Prices and Mortgage Defaults: Econometric Models and Risk Management Applications

Xiangjing Wei

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House Prices and Mortgage Defaults: Econometric Models and Risk Management Applications

BY

XIANGJING WEI

A Dissertation Submitted in Partial Fulfillment of the Requirements for the Degree
of
Doctor of Philosophy
in the Robinson College of Business
of
Georgia State University

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ACCEPTANCE

This dissertation was prepared under the direction of the candidate's Dissertation Committee. It has been approved and accepted by all members of that committee, and it has been accepted in partial fulfillment of the requirements for the degree of Doctor in Philosophy in Business Administration in the Robinson College of Business of Georgia State University.

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ABSTRACT

House Prices and Mortgage Defaults: Econometric Models and Risk Management Applications

By

XIANGJING WEI

07/2010

Committee Chair: Dr. _SHAUN WANG___

Major Department: __Risk Management and Insurance______

This dissertation first investigates the possible house price trend and the relationship with the mortgage market, from the perspective of risk management; then it chooses the angle from bond insurers and figures out possible methods to avoid capital procyclicality.

As for the first chapter, based on the Granger-Causality test, we apply vector auto regression models (VAR) and simultaneous equations models (SEM) to estimate the dynamic relations among house price returns, mortgage rates and mortgage default rates, using historical data during the time period of 1979 through second quarter 2008. We find that house prices would be better estimated and predicted with the consideration of the mortgage market.

In Chapter II, following the methodology of co-integration, we first construct four succinct measures to display the possible intrinsic values of house prices. The fifth measure is defined as a weighted average of the first four. In the short run, house price return dynamics are investigated by dynamic adjustments following Capozza et al (2002) and error correction models. The estimations reflect gradual adjustments towards the long-run intrinsic values. The

impacts of the mortgage credit market on house price returns are also analyzed. Furthermore, we examine the possible overshooting problem of house price returns. By both analytical derivations and simulations, we demonstrate the effects of the coefficients on overshooting.

In Chapter III, we adopt a structural model with time-varying correlations, which are closely tied up with the business cycle, for bond insurers. When deriving the total loss distribution and economic capital for a bond insurer, we consider losses due to bond insurers' downgrading and losses from both insurance contracts and investment portfolio. On that basis, we propose forward-looking smoothing rules of capital over a full business cycle, instead of only based on a short-term horizon, to avoid the procyclicality. With the smoothed capital, a bond insurer can actually establish some capital buffer in good times to support the potential losses in crisis.

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Chapter 0: Introduction

U.S. house prices peaked in 2006 and began their steep decline thereafter. Mortgage defaults and foreclosure soared. As a result, mortgage-backed securities lost most of their value. Since the securities are widely held by financial firms, there has been a large decline in the capital of many financial institutions, including bond insurers.

There are two main purposes of this dissertation. The first is to investigate the possible house price trend and the relationship with the mortgage market, from the perspective of risk management. We are not trying to analyze the underlying economic mechanisms of the crisis. Instead, we study the problem from the viewpoint of a professional in risk management. Many companies, especially financial institutions, hold products related with mortgages. They need to have a rough idea about the state of the housing market, whether mortgage defaults have impacts on the housing market, and what the corresponding risk management methods are. By our analysis, we can at least answer the following questions: With the consideration of mortgage defaults, will the house prices be better estimated and predicted? How can we estimate the intrinsic values of house prices? Will the house prices adjust back immediately if they are overvalued or undervalued?

The second purpose is linked to risk management applications. More specifically, we consider a bond insurer and analyze the impacts of the business cycle (especially financial crisis) on its financial situations. Bond insurers are very sensitive to the changes in the business cycle. In particular, the bonds have the tendency to have higher correlations with each other in deep recession (or crisis), so systemic economic shocks may have serious impacts on bond insurers. In order to keep their ratings, bond insurers need more capital in credit crisis, which may worsen their financial conditions further. Therefore, their capital requirements may be even more pro-cyclical. Based on our model, we propose possible methods to avoid capital procyclicality.

Chapters I and II are related with the first purpose. More precisely, in Chapter I, we analyze the interactions among house price returns, default rates and mortgage interest rates. We utilize a Vector Autoregressive model (VAR) and a Simultaneous Equation model (SEM). Based on the observations and our analysis, house prices impact mortgage defaults, and *vice versa*. This could explain why the severity of the crisis is heavier than what we originally expected. By working on a structural model, the dynamic relations among house price returns, mortgage rates and default rates can be investigated better. However, the relatively high standard errors may render the estimates fluctuating in a wide range.

In Chapter II, we examine the long term housing intrinsic value, short term dynamics, and overshooting problems. The basic method we employ to detect the housing intrinsic value is co-integration. We define five measures of housing intrinsic values considering different economic aspects as the possible criteria in order to determine whether house prices are overvalued or undervalued. After determining the intrinsic values of house prices, we continue to investigate the short term dynamics based on dynamic adjustments following Capozza *et al*. (2002) and error correction models. Specifically, the serial correlation term (momentum effect) and the deviation term from the intrinsic value are examined. As for the possible overshooting problem of house price returns, we analytically derive the consecutive house price returns after an unexpected shock at time 0 assuming the intrinsic house price returns equal to zero. Then, by simulation, we demonstrate the possible effects of the coefficients of the serial correlation term, deviation term and immediate adjustment term.

Chapter III serves the second purpose. More specifically, we adopt a structural model with time-varying correlations, which are closely tied up with the business cycle, for bond insurers. We analyze the impacts of bond insurers' downgrades on bond values and incorporate them into the total losses of bond insurers. The losses on both insurance contracts and investment portfolios are also considered. On that basis, we propose forward-looking smoothing rules of capital over a full business cycle, instead of only based on a short-term horizon, to avoid the procyclicality. The simulation results show the effects of changing parameter values in smoothing rules. The smoothed capital may vary from lower degree of procyclicality to totally counter-cyclicality, corresponding to the different parameter values. With the smoothed capital, a bond insurer can actually establish some capital buffer in good times to support the potential losses in times of crisis.

Chapter I: Dynamic Relationships among House Price Returns, Mortgage Rates and Default Rates: Study of the Recent Mortgage Crisis

Abstract

Based on the Granger-Causality test, we apply vector auto regression models (VAR) and simultaneous equations models (SEM) to estimate the dynamic relations among house price returns, mortgage rates and mortgage default rates, using historical data during the time period of 1979 through second quarter 2008. We estimate that, holding all other factors constant, two consecutive 1% increases of default rates can drive house price returns down by about 7%-18%. Conversely, two consecutive 1% decreases of house price returns can increase the current default rate by 0.04%-0.09%.

We apply our econometric models in making predictions using data up to the second quarter of 2008. The FHFA's and Case-Shiller's indices exhibit different patterns and thus they yield different predictions. The predicted future level of the FHFA's house price returns will remain negative and reach the lowest value in 2010; it may take some years for the house price returns to become positive. However, we get more optimistic forecasts using the Case-Shiller's index, whereas the future house price returns would become positive since 2010, and mortgage default rates will peak by 2010 and decrease thereafter. We add caveats for interpreting these mechanical forecasts: they do not reflect many important dynamics that will strongly impact the housing markets, for instance, the various government mortgage modification programs, and the inventory of excess housing units.

I.1. Introduction

The mortgage crisis began with sharp falls of house prices after the United States housing boom peaked in 2005-2006 and became apparent in 2007, signaled by a sharp rise in mortgage defaults and foreclosures in the United States. It has affected almost all investments which derive their values from mortgage loans, such as Mortgage-Backed Securities (MBSs) and Collateralized Debt Obligations (CDOs). Up to the middle of 2008, several hundred billion dollars in losses have been reported and written off by the financial institutions that were heavily invested in those products. Much of the capital of the banking system was wiped out. Mortgage insurers and bond insurers were also hit hard. Further, the mortgage crisis spread to the general economy.

Many papers discuss the causes and effects of the mortgage crisis. Their analyses focus on many aspects: falling house prices, high-risk mortgage loans and eased lending standards, securitization, roles of credit rating companies, government policies and so on. For example, Crouhy *et al.* (2008) examine the players and issues at the heart of this crisis. Cagan (2007) and Weaver and Reeves (2007) emphasize the impacts of mortgage rate reset. Mayer *et al.* (2009) find that the rise in defaults through mid-2008 was not linked to the novel mortgage products. Foote *et al.* (2008) state that interest-rate resets may not be the main problem in the mortgage market. Both of the two papers claim that higher foreclosure rates stem from falling house prices. Demyanyk and Van Hemert (2010) analyze loan-level data and find that the quality of loans deteriorated for six consecutive years before the crisis. Problems could have been detected long before the crisis, but they were masked by high house price appreciation between 2003 and 2005. Greenlaw *et al.* (2008) and Hatzius (2008) put emphasis on modeling mortgage credit losses, based on the effects of home price declines on foreclosure and mortgage credit losses.

Taylor (2008) investigates the role of government actions and interventions in the financial crisis and claims:

(1) The main cause of the housing boom and the resulting bust is monetary excesses, in terms of the low federal funds interest rates from 2000 to 2004.

(2) Falling house prices cause higher mortgage delinquencies and foreclosures.

(3) The above effects are amplified by the excessive risk taking behaviors, such as the use of subprime mortgages, the adjustable rate mortgage variety, the easy mortgage underwriting procedures, securitization, etc.

We analyze the interactions among house price returns, default rates and mortgage interest rates. Our purpose is to at least answer the question: With the consideration of mortgage defaults, will the house prices be better estimated and predicted?

We utilize a Vector Autoregressive model (VAR) and a Simultaneous Equation model (SEM). Based on the observations and our analysis, house prices impact mortgage defaults, and *vice versa*. This could explain why the severity of the crisis is heavier than what we originally expected. Relying on a structural model, the dynamic relations among house price returns, mortgage rates and default rates can be investigated explicitly. Holding all other factors constant, two consecutive 1% increases of default rates can drive the FHFA's[1] house price returns down by about 7.64% and the Case-Shiller's current house price return down by about 18%. Conversely, two consecutive 1% decreases of the FHFA's or Case-Shiller's house price returns can drag the current

[1] The Federal Housing Finance Agency. It is previously called "the Office of Federal Housing Enterprise Oversight (OFHEO)"

default rate up by 0.09 percent or 0.04 percent, respectively. However, relatively high standard errors may render the above estimates fluctuating in a wide range.

We apply our models in making predictions using data up to the second quarter of 2008. The FHFA's and Case-Shiller's indices exhibit different patterns and thus they yield different predictions as well. Per the data up to the second quarter of 2008, the expected future level of the FHFA's house price returns will remain negative and reach the lowest value in 2010; it may take some years for the house price returns to become positive. However, we get more optimistic forecasts using the Case-Shiller's index: the future house price returns tend positive beyond 2010; and mortgage default rates peak by 2010 and decrease thereafter.

The structure of the rest of this chapter is as follows: Section I.2 explains the historical dynamics and data. Section I.3 applies the Granger-Causality test. Sections I.4 presents the VAR process. Section I.5 presents the Simultaneous Equation Model and extended SEM model. Section I.6 makes predictions based on the models. Section I.7 summarizes our conclusions.

I. 2. Data

For house prices, we investigate both the FHFA's House Price Index and the S&P/ Case-Shiller's House Price Index.[2] Both of the indices are repeat sales indices. The S&P/

[2] Additionally, the current house price series (or indices) used to measure national trends include *the median price of existing homes sold* (published by the National Association of Realtors) and *the median price of new homes sold* (published by the Bureau of the Census of the U.S. Department of Commerce). These two indices are not seasonally adjusted and reflect only recent sales, so they are volatile in the short run.

Case-Shiller index is value-weighted, based on 10 or 20 metropolitan areas[3], available from 1987. The FHFA's index is unit-weighted, based on the fifty states and Washington D.C., available from 1975. Moreover the FHFA's House Price Index only uses the data based on Fannie Mae and Freddie Mac mortgages. The Case-Shiller's House Price Index obtains data from county assessor and recorder offices, and therefore covers more houses in the specific areas.

Figure I.1 displays the differences between the two indices. The Case-Shiller's Index shows larger fluctuations than the FHFA's Index. Between the end of year 2006 and the second quarter of 2008, the FHFA's house price index had a cumulative decrease of −1.16 percent, while the Case-Shiller's house price returns accumulated to a loss of about −20 percent.

As for the mortgage rate, we use the 30-year fixed mortgage rates from the Federal Reserve's website.

Although it is generally agreed that this mortgage crisis originated in subprime adjustable rate mortgages (ARM), the mortgage default rates for all the listed types of mortgage products have been increasing since late 2006. Figure I.2 shows the percentages of different loans[4] past due 90 days between 1998 and the second quarter of 2008, obtained from the Mortgage Bankers Association. Table I.1 displays different mortgage delinquency rates and foreclosure rates in the year end 2005-2007 and the second quarter

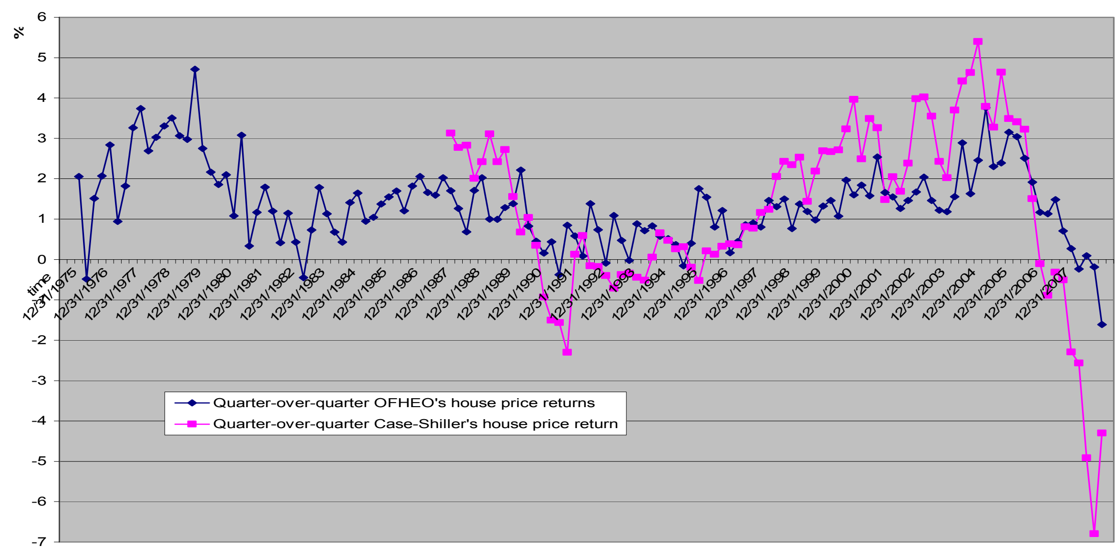
[3] The 10 metropolitan areas include Boston, Chicago, Denver, Las Vegas, Los Angeles, Miami, New York, San Diego, San Francisco, and Washington DC. The 20 metropolitan areas also include Atlanta, Charlotte, Cleveland, Dallas, Detroit, Minneapolis, Phoenix, Portland (Oregon), Seattle, and Tampa

[4] They include Prime FRM (fixed rate mortgages), Prime ARM (adjustable rate mortgages), Subprime FRM, Subprime ARM and all loans.

of 2008, which rose at different rates since 2007, even for Prime mortgages. We use the percentage of all loans past due 90 days as a proxy for the mortgage default rate[5].

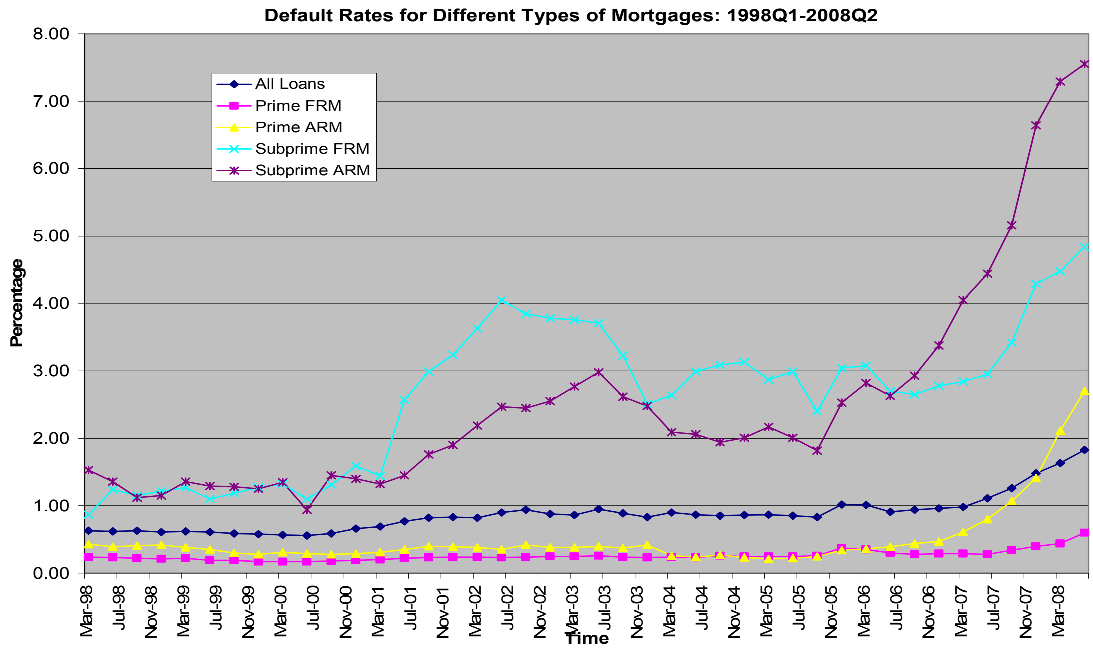
Figure I.3 shows the historical data in the FHFA's house price return, mortgage interest rates and default rates over time.

Figure I.1: quarter-over-quarter the FHFA's vs. Case-Shiller's house price returns to 2008Q2



[5] Practically, the differences between mortgage delinquency and mortgage default are based on the number of days of missed installments. Delinquency refers to the non-payment of a mortgage payment due, so it may be defined as a 30-days-and-over delinquency, a 60-days-and-over delinquency or a 90-days-and-over delinquency. Default happens when a borrower fails to pay back 90-days' installment due and the fourth payment is due.

Figure I.2: default rates for all types of mortgage loans



Source: Mortgage Bankers Association

Figure I.3: house price returns, mortgage rates and default rates

This figure displays the historical quarterly data of house price returns, mortgage rates and default rates from first quarter 1979 to second quarter 2008. The left-side y-axis is for house price return and default rate. The right-side y-axis is for mortgage rate. The default rates are multiplied by 2 in this graph to keep them in the same range as the house price returns.

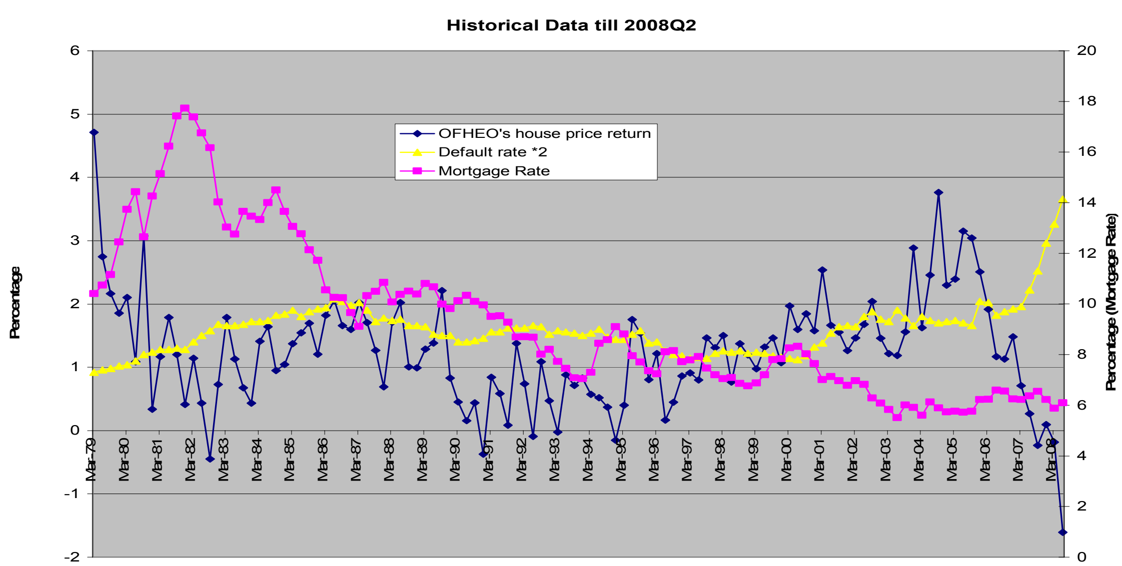


Table I.1: Mortgage Delinquency Rates

The default rates of all types of mortgages have increased in different degrees, especially the adjustable rate mortgages (ARM).

		2005Q4	Increased*	2006Q4	Increased*	2007Q4	Increased*	2008Q2	Increased**
All loans	Loans Past Due 30 Days	2.85%	2.89%	3.08%	8.07%	3.20%	3.90%	3.30%	3.13%
	Loans Past Due 60 Days	0.83%	10.67%	0.90%	8.43%	1.15%	27.78%	1.28%	11.30%
	Loans Past Due 90 Days	1.02%	18.60%	0.96%	-5.88%	1.48%	54.17%	1.83%	23.65%
	Loans in Foreclosure	0.42%	-8.70%	0.54%	28.57%	0.83%	53.70%	1.19%	43.37%
Prime FRM Loans	Loans Past Due 30 Days	1.49%	0.00%	1.64%	10.07%	1.72%	4.88%	1.90%	10.47%
	Loans Past Due 60 Days	0.35%	12.90%	0.34%	-2.86%	0.44%	29.41%	0.57%	29.55%
	Loans Past Due 90 Days	0.37%	48.00%	0.29%	-21.62%	0.40%	37.93%	0.60%	50.00%
	Loans in Foreclosure	0.15%	-11.76%	0.16%	6.67%	0.22%	37.50%	0.37%	68.18%
Prime ARM Loans	Loans Past Due 30 Days	1.76%	13.55%	2.30%	30.68%	2.89%	25.65%	3.24%	12.11%
	Loans Past Due 60 Days	0.44%	33.33%	0.63%	43.18%	1.20%	90.48%	1.56%	30.00%
	Loans Past Due 90 Days	0.34%	47.83%	0.47%	38.24%	1.41%	200.00%	2.70%	91.49%
	Loans in Foreclosure	0.20%	5.26%	0.41%	105.00%	1.06%	158.54%	1.93%	82.08%
Subprime FRM Loans	Loans Past Due 30 Days	5.06%	1.00%	5.57%	10.08%	7.17%	28.73%	8.05%	12.27%
	Loans Past Due 60 Days	1.60%	1.27%	1.73%	8.12%	2.54%	46.82%	3.14%	23.62%
	Loans Past Due 90 Days	3.04%	-2.88%	2.78%	-8.55%	4.29%	54.32%	4.84%	12.82%
	Loans in Foreclosure	1.05%	-23.36%	1.09%	3.81%	1.52%	39.45%	2.28%	50.00%
Subprime ARM Loans	Loans Past Due 30 Days	6.74%	13.66%	7.93%	17.66%	8.80%	10.97%	8.68%	-1.36%
	Loans Past Due 60 Days	2.35%	23.68%	3.13%	33.19%	4.58%	46.33%	4.80%	4.80%
	Loans Past Due 90 Days	2.53%	25.87%	3.38%	33.60%	6.64%	96.45%	7.55%	13.70%
	Loans in Foreclosure	1.55%	3.33%	2.70%	74.19%	5.29%	95.93%	7.09%	34.03%

Sources: Mortgage Bankers Association
*: Increased percentage compared with one year ago
**: Increased percentage compared with the end of 2007

I.3. Granger Causality test

We first apply the Granger Causality test[6], which examines whether one time series can help forecast another variable, to support the dynamic analysis among house price returns, mortgage rates and default rates.

Suppose we have two terms X and Y, which are time series variables. In the presence of lagged Y, if an F-test on lagged observations of X shows that the X observations provide statistically significant information about future values of Y, then X is said to Granger-cause Y. Therefore, Y is better predicted per the histories of X and Y than only per the history of Y. The VAR model is a simple approach to implementing the Granger Causality test.

Table I.2 exhibits the results of the test by using the data through the second quarter of 2008. We obtain similar results for both the FHFA's and Case-Shiller's house price returns. At the 10% significance level, house price returns Granger-cause default rates, and vice versa. House price returns Granger-cause mortgage interest rates, but conversely not. There is no apparent Granger causality between mortgage interest rates and default rates. The reason may be that our test is based on the aggregate level data. However, the results are sufficient for us to investigate the dynamic relationships among the three variables, especially between house price returns and default rates.

[6] The detailed explanation can be found in Granger (1969).

Table I.2: Granger Causality Test for house price returns, default rates and mortgage rates

The table shows whether group 2 variable Granger-causes group 1 variable. The null hypothesis of the test is that group 2 variable does not Granger-cause group 1 variable.

We obtain similar results for both the FHFA's and Case-Shiller's house price returns. At the 10% significance level, house price returns Granger-cause default rates, and vice versa. House price returns Granger-cause mortgage interest rates, but conversely not. There is no Granger causality between mortgage interest rates and default rates.

Granger Causality Test				
Test	Group 1 Variable	Group 2 Variable	Chi-Square	Prob>Chisq
1	FHFA House Price Return	First-differenced Default Rate	11.38	0.0226
2	FHFA House Price Return	First-differenced Mortgage Rate	5.19	0.2683
3	First-differenced Default Rate	FHFA House Price Return	8.41	0.0775
4	First-differenced Default Rate	First-differenced Mortgage Rate	0.79	0.9395
5	First-differenced Mortgage Rate	FHFA House Price Return	10.39	0.0343
6	First-differenced Mortgage Rate	First-differenced Default Rate	0.59	0.9636
Test	Group 1 Variable	Group 2 Variable	Chi-Square	Prob>Chisq
1	Case-Shiller's House Price Return	First-differenced Default Rate	41.06	<.0001
2	Case-Shiller's House Price Return	First-differenced Mortgage Rate	8.38	0.3005
3	First-differenced Default Rate	Case-Shiller's House Price Return	18.37	0.0104
4	First-differenced Default Rate	First-differenced Mortgage Rate	5.97	0.5438
5	First-differenced Mortgage Rate	Case-Shiller's House Price Return	12.31	0.0907
6	First-differenced Mortgage Rate	First-differenced Default Rate	2.7	0.9112

I.4. VAR process

We first utilize a vector autoregressive process VAR(p):

$$Y_t = \sum_{i=1}^{p} \Phi_i Y_{t-i} + \varepsilon_t ,$$

where $Y_t = (HR_t, D_t, MR_t)'$ refers to the endogenous variables and $\varepsilon_t = (\varepsilon_{1t}, \varepsilon_{2t}, \varepsilon_{3t})'$ refers to a vector white noise process. Φ_i is a 3×3 matrix. HR_t is quarter-over-quarter house price return at time *t*; MR_t denotes the mortgage rate at time *t*; D_t refers to the default rate at time *t*. The mortgage rates and default rates are first-differenced since they are integrated of order one.

According to the Akaike information criterion (AIC)[7], we choose VAR(7) for the FHFA's house price return and VAR(12) for the Case-Shiller's house price return, with the first-differenced mortgage interest rates and default rates. The lagged periods are chosen based on the significance level of the coefficients.

I.5. Simultaneous Equation Model (SEM) and Extended SEM

I.5.1. Simultaneous Equation Model (SEM)

Case and Shiller (1990) build a forecasting model for house price return, including both lagged house price returns and other exogenous variables.

In our paper, since we have more than one endogenous variable, a simultaneous equations model is introduced and some exogenous variables are included. Our model can be represented as

[7] Akaike (1974)

$$HR_t = f_1\left(\sum_{s=0}^{p_2} MR_{t-s}, \sum_{s=0}^{p_3} D_{t-s}, X\right) + \sum_{s=1}^{p_1} a_s HR_{t-s} + \varepsilon_1,$$
$$MR_t = f_2\left(\sum_{s=0}^{p_1} HR_{t-s}, \sum_{s=0}^{p_3} D_{t-s}, Y\right) + \sum_{s=1}^{p_2} b_s MR_{t-s} + \varepsilon_2, \quad \text{(I.1)}$$
$$D_t = f_3\left(\sum_{s=0}^{p_1} HR_{t-s}, \sum_{s=0}^{p_2} MR_{t-s}, Z\right) + \sum_{s=1}^{p_3} c_s D_{t-s} + \varepsilon_3,$$

where X, Y and Z refer to vectors of economic variables. The first part of each equation can be regarded as a fundamental value or an intrinsic value of the endogenous variable. The serial correlation part represents the momentum. This model is under a structural framework, so that the relationships among the variables could be more clearly examined.

I.5.2 SEM: Model Specification

The Simultaneous Equation Model can be specified as follows:

$$HR_t = \alpha_0 + \alpha_1 MR_t + \alpha_2 MR_{t-1} + \sum_{s=1}^{p_2} b_s^1 \Delta MR_{t-p_3} + \alpha_3 D_t + \alpha_4 D_{t-1} + \sum_{s=1}^{p_3} c_s^1 \Delta D_{t-p_3}$$
$$+ \alpha_5 Inf_t + \alpha_6 Inf_{t-1} + \alpha_7 \Delta CC_{t-1} + \alpha_8 \Delta Inc_t + \alpha_9 Unem_t + \alpha_{10} \Delta Unem_t + \alpha_{11} \Delta Tb3m_t$$
$$+ \sum_{s=1}^{p_1} a_s^1 HR_{t-s} + \varepsilon_1 \quad \text{(I.2a)}$$

$$MR_t = \beta_0 + \sum_{s=0}^{p_1} a_s^2 HR_{t-s} + \beta_1 D_t + \beta_2 D_{t-1} + \sum_{s=1}^{p_3} c_s^2 \Delta D_{t-s} + \beta_3 Inf_{t-1} + \beta_4 \Delta GDP_t$$
$$+ \beta_5 \Delta TB_t + \beta_6 \Delta TB3m_t + \beta_7 MR_{t-1} + \sum_{s=1}^{p_2} b_s^2 \Delta MR_{t-s} + \varepsilon_2 \quad \text{(I.2b)}$$

$$D_t = \gamma_0 + \sum_{s=0}^{p_1} a_s^3 HR_{t-s} + \gamma_1 MR_t + \gamma_2 MR_{t-1} + \sum_{s=1}^{p_2} b_s^3 \Delta MR_{t-s} + \gamma_3 Inf_{t-1} + \gamma_4 CLTV_t$$
$$+ \gamma_5 \Delta CLTV_t + \gamma_6 \Delta Inc_t + \gamma_7 \Delta TB3m_t + \gamma_8 D_{t-1} + \sum_{s=1}^{p_3} c_s^3 \Delta D_{t-s} + \varepsilon_3 \quad \text{(I.2c)}$$

In the house model (I.2a), we use the *inflation rate (Inf)*[8], *disposable personal income (Inc)*[9], *unemployment rate (Unem)*[10], *construction cost (CC)*[11], and the *3-month Treasury bill rate (TB3m)*[12] as the exogenous variables. We choose the *3-month Treasury bill rate* as the indicator of market interest rate.

The mortgage rate equation (I.2b) includes *inflation rate (Inf), gross domestic product (GDP)*[13], *10-year treasury bond rate (TB), the 3-month Treasury bill rate (TB3m).*

The default rate equation (I.2c) may contain the *inflation rate (Inf), thecomposite loan-to-value ratio (CLTV)*[14], *disposable personal income (Inc),* and *the 3-month Treasury bill rate (TB3m).*

We checked the stationarity of all the variables, using the Augmented Dickey Fuller (ADF) test. Only house price return and inflation rate reject the non-stationary null at 1% significance level. All other variables are integrated of order 1. In order to avoid spurious regression, we correspondingly add the lagged or differenced terms.

To detect multicollinearity, we examined *tolerance*[15], *variance inflation factor*[16] and *condition indexes*[17]. The variables currently in equation I.2a-I.2c show no serious multicollinearity.

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[8]The data of inflation rate come from the Consumption Price Index from the U.S. Bureau of Labor Statistics (BLS)

[9] The data on disposable personal income are from the Bureau of Economic Analysis' website

[10] The unemployment rate is from the U.S. Bureau of Labor Statistics (BLS) Household Survey

[11] We use Construction Price Index as the measure of *construction cost*. This index is the price deflator index of new one-family houses under construction from U.S. Census Bureau.

[12]3-month Treasury bill rate and 10-year Treasury bond rate are available on the Federal Reserve Board's website

[13] from the Bureau of Economic Analysis (BEA)

[14] from the U.S. Federal Housing Finance Board

[15] The *tolerance* measures the correlation between one independent variable and all the other independent variables. If we define $R^2_{X,\tilde{X}}$ as the correlation between one dependent variable X and all the other independent variables $\tilde{X}$, then the *tolerance (TOL)* would be $TOL_X = 1 - R^2_{X,\tilde{X}}$. A small value of tolerance means that the variable X is highly correlated with the other variables.

For the models dealing with the FHFA's house price returns, we use quarterly data from first quarter 1979 through second quarter 2008, with 128 observations in total, due to the data source restrictions of mortgage default rate. For the models dealing with the Case-Shiller's house price returns, we use quarterly data from first quarter 1987 through second quarter 2008, with 86 observations in total.

I.5.3 SEM: Estimation Results

Using the three-stage-least-squares method, we carry out two regressions for both the FHFA's and Case-Shiller's house price returns. For house price returns, mortgage rates and default rates, one regression contains only one-period-lagged observations, while the other one includes multi-period-lagged or multi-period-changed observations. The results are listed in Table I.3. Since for the exogenous variables, the coefficients either have the same signs as we expected or are insignificant, our analysis mainly focuses on the three key variables.

Serial Correlation Term

All the three endogenous variables have highly significantly positive serial correlation coefficients. Obviously, they have a strong tendency to keep their original values.

Interrelationship among the three variables

1) House price return equation (Table I.3(1))

---

[16] The *variance inflation factor (VIF)* is the inverse of tolerance, $VIF_x = 1 / TOL_x$, showing the degree by which the standard error of the estimator is inflated by multicollinearity. Practically, $TOL < 0.1$ and equivalently $VIF > 10$ indicate a multicollinearity problem.

[17]The *Conditional index* is the ratio of a specific eigenvalue over the maximum of all eigenvalues of the model matrix. As an informal rule, a conditional index over 30 may show multicollinearity.

Default rates have consistent effects on house price returns for both regressions and both indices. The current default rate has negative coefficients on house price returns, showing that the increased default rate will drive the current house price returns down, due to increased supply or shrunken credit. Combining this observation with the positive estimates of the lagged default rates would reflect a complicated process. Take regression 1 for the FHFA's house price return as an example. The effects of a 1% increase of one-period-lagged default rate on the current house price return are a 9.56% [18] increase. Additionally, if the current default rate also increases by 1%, then the current house price return will decrease by 7.64% (=17.20%-9.56%) finally. Similarly, the two consecutive 1-percent increases in default rates will depress the Case-Shiller's current house price return by 18.37%.

As for mortgage rates, the dominant estimates are negatively correlated with the current house price return, meaning that low mortgage rates will drive the housing demand up and so increase the house price returns, and *vice versa*. The two consecutive 1% increases of mortgage rates will depress the FHFA's or Case-Shiller's current house price returns by 0.47% or 0.83% respectively.

2) Mortgage rate equation (Table I.3(2))

The impact of the default rate on the mortgage rate is complicated, because mortgage rates may be decomposed into two parts. One is market interest rate and the other is margin, which reflects the willingness of the banks to provide credit. Due to the margin effect, although the market interest rates fell since September 2007, the mortgage rates did not change in the same

[18] In regression 1 for the FHFA's house price return, a 1% increase of one-period-lagged default rate will lead to a 10.77% decrease in one-period-lagged house price returns and correspondingly a 5.82%(=10.77%*0.54) decrease in current house price returns. At the same time, the 1% increase of one-period-lagged default rate will result in a 11.42% increase in current house price returns. Therefore, the net effects of a 1% increase of one-period-lagged default rate on the current house price return would be a 5.60% (=11.42%-5.82%) increase in current house price return.

direction or similar magnitude. The increase in default rates commonly leads to declining market interest rates due to the policymakers' intervention and increased margin due to the unwillingness for the mortgage providers to provide credit. The coefficients on default rates with the two house price indices are different, reflecting the complicated effects.

As for house price returns, the current house price returns are dominant and have negative effects. It means that higher house price return will urge the mortgage providers to provide more credit, and so ease the credit market and lower the mortgage rate. For example, the two consecutive 1% increases of the FHFA's or Case-Shiller's current house price returns will depress mortgage rates by 0.61% or 0.07% respectively.

3) Default rate equation (Table I.3(3))

The negative coefficient on the current house price return shows that the dropped house price return lowers the housing equity and makes it more difficult to pay back the mortgage by refinancing, which drives the default rate up.

Again take Regression 1 as example. Two consecutive 1% decreases of the FHFA's house price returns will push the current default rate up by 0.09 percent. And two consecutive 1% decreases of the Case-Shiller's house price returns will push the current default rate up by 0.04 percent.

In terms of mortgage rates, for the new mortgagors, the augmented fixed mortgage rates make it more difficult for buyers with lower affordability to get a mortgage loan, which causes lower default rate. At the same time, the current mortgagors have comparatively lower contract rates and tend to keep their contracts and not to default. The results may not be consistent with the intuition that the risen mortgage rates drive up default rates. The reasons may be that (1) we are analyzing at the aggregate level; (2) by using 30-year fixed mortgage rate, we exclude the adjustment of the mortgage rates in the current contracts.

Table I.3 (1): regression results for SEM –house price return equation

This table exhibits the regression results of the Simultaneous Equation Model (SEM) for the **house price return equation.** The part (1) is from the **model with the FHFA's house price returns**. The part (2) is from the **model with the Case-Shiller's house price returns**. The data are centered at the mean.

Equation: House Price Return	**With the FHFA's House Price Return (1)**		**With the Case-Shiller's House Price Return(2)**	
Variable	Regression 1	Regression 2	Regression 1	Regression 2
Intercept	0.026 (0.200)	0.076 (0.310)	-0.642 (0.658)	-0.503 (0.336)
1-period-lagged house price return	0.504** (0.104)	0.402** (0.228)	0.707** (0.272)	1.088** (0.175)
2-period-lagged house price return		0.112 (0.257)		-0.523** (0.272)
3-period-lagged house price return		0.114 (0.328)		0.360* (0.252)
4-period-lagged house price return	--	--		-0.288* (0.216)
30-year fixed mortgage rate	-0.772** (0.240)	-0.856** (0.418)	-0.527 (1.249)	-0.338 (0.497)
1-period-lagged 30-year fixed mortgage rate	0.688** (0.226)	0.767** (0.400)	0.066 (0.837)	0.001 (0.404)
1-period-lagged change of 30-year fixed mortgage rate		-0.094 (0.343)		-0.509* (0.382)
2-period-lagged change of 30-year fixed mortgage rate		0.271 (0.369)		-0.213 (0.309)
3-period-lagged change of 30-year fixed mortgage rate		0.016 (0.262)		0.278 (0.334)
default rate	-17.201** (7.037)	-18.255 (18.530)	-25.596 (33.181)	-23.263** (12.027)
1-period-lagged default rate	18.232** (7.705)	18.383 (19.578)	25.173 (34.236)	23.627** (13.190)
1-period-lagged change of default rate		4.543 (4.630)		1.413 (2.872)
2-period-lagged change of default rate		0.539 (2.718)		-2.296 (2.969)
3-period-lagged change of default rate		4.413 (6.371)		4.070* (2.881)
inflation rate	-0.044 (0.103)	0.014 (0.109)	-0.008 (0.343)	-0.343 (0.269)
lagged inflation rate	0.593* (0.225)	0.462 (0.514)	0.944 (1.767)	0.807 (0.647)
change in income	6.648 (9.777)	-1.842 (11.418)	25.028 (38.340)	6.725 (15.026)
change of 3-month Treasury bill rate	0.069 (0.121)	0.096 (0.230)	-0.462 (0.569)	0.469 (0.424)
change in construction cost	0.044 (8.021)	2.617 (9.579)	6.727 (34.194)	2.129 (16.539)
unemployment rate	-0.003 (0.047)	0.030 (0.104)	0.069 (0.374)	-0.273 (0.323)
change in unemployment rate	0.069 (0.384)	-0.186 (0.657)	-0.389 (1.815)	1.401 (1.096)

Note: *20%, **10% indicate the corresponding significance levels. The numbers in parentheses refer to the standard errors of the coefficients.

Table I.3(2): regression results for Model 2—Mortgage Rate Equation

This table exhibits the regression results of the Simultaneous Equation Model (SEM) for **the mortgage rate equation**. Part (1) is from the **model with the FHFA's house price returns**. Part (2) is from the **model with the Case-Shiller's house price returns**. The data are centered at the mean.

Equation: Mortgage Rate	**With the FHFA's house price return (1)**		**With the Case-Shiller's house price return (2)**	
Variable	Regression 1	Regression 2	Regression 1	Regression 2
Intercept	-0.002 (0.102)	0.008 (0.108)	-0.157** (0.065)	-0.103 (0.068)
1-period-lagged 30-year fixed mortgage rate	0.963** (0.014)	0.957** (0.018)	0.992** (0.021)	0.998** (0.019)
1-period-lagged change of 30-year fixed mortgage rate		0.075 (0.100)		0.018 (0.054)
2-period-lagged change of 30-year fixed mortgage rate		0.084 (0.100)		-0.007 (0.053)
3-period-lagged change of 30-year fixed mortgage rate		-0.045 (0.073)		0.031 (0.050)
default rate	-3.807* (2.586)	-7.617** (3.934)	1.360* (1.071)	1.123 (0.896)
1-period-lagged default rate	3.832* (2.793)	7.595** (4.057)	-1.566* (1.081)	-1.319* (0.910)
1-period-lagged change of default rate		2.180** (1.197)		0.592* (0.418)
2-period-lagged change of default rate		-0.218 (0.855)		0.240 (0.406)
3-period-lagged change of default rate		1.299 (1.455)		0.111 (0.433)
house price return	-0.477** (0.149)	-0.616** (0.179)	-0.064 (0.073)	-0.028 (0.058)
1-period-lagged house price return	0.325** (0.084)	0.367** (0.086)	0.059 (0.063)	0.051 (0.069)
2-period-lagged house price return		0.003 (0.081)		-0.025 (0.046)
3-period-lagged house price return		0.088 (0.084)		0.024 (0.043)
4-period-lagged house price return	--	--		-0.026 (0.027)
lagged inflation rate	0.241** (0.087)	0.245** (0.115)	-0.011 (0.062)	0.023 (0.060)
Change of nominal GDP	0.917 (5.440)	0.955 (5.718)	8.867** (4.091)	6.784* (4.327)
Change of 10-year Treasury bond rate	0.567** (0.096)	0.443** (0.111)	0.820** (0.052)	0.825** (0.053)
change of 3-month Treasury bill rate	-0.027 (0.055)	0.009 (0.066)	0.063* (0.047)	0.071 (0.059)

Note: *20%, **10% indicate the corresponding significance levels. The numbers in parentheses refer to the standard errors of the coefficients.

Table I.3 (3): regression results for Model 2—Default Rate Equation

This table exhibits the regression results of the Simultaneous Equation Model (SEM) for **the default rate equation**. Part (1) is from the **model with the FHFA's house price returns**. Part (2) is from the **model with the Case-Shiller's house price returns**. The data are centered at the mean.

Equation: Default Rate	**With the FHFA's house price return (1)**		**With the Case-Shiller's house price return(2)**	
Variable	Regression 1	Regression 2	Regression 1	Regression 2
Intercept	0.002 (0.011)	0.006 (0.009)	-0.023* (0.015)	-0.015 (0.016)
1-period-lagged default rate	1.059** (0.034)	1.019** (0.038)	0.994** (0.054)	1.056** (0.099)
1-period-lagged change of default rate		0.238** (0.117)		0.074 (0.165)
2-period-lagged change of default rate		0.023 (0.114)		-0.070 (0.145)
3-period-lagged change of default rate		0.257** (0.126)		0.202* (0.143)
30-year fixed mortgage rate	-0.046** (0.017)	-0.040** (0.016)	-0.022 (0.022)	-0.019 (0.023)
1-period-lagged 30-year fixed mortgage rate	0.041** (0.016)	0.036** (0.016)	0.005 (0.021)	0.007 (0.023)
1-period-lagged change of 30-year fixed mortgage rate		-0.007 (0.012)		-0.014 (0.019)
2-period-lagged change of 30-year fixed mortgage rate		0.014* (0.011)		0.006 (0.021)
3-period-lagged change of 30-year fixed mortgage rate		-0.001 (0.010)		0.004 (0.020)
house price return	-0.058** (0.019)	-0.043** (0.016)	-0.033* (0.025)	-0.006 (0.032)
1-period-lagged house price return	0.029** (0.012)	0.017* (0.012)	0.022 (0.022)	0.005 (0.034)
2-period-lagged house price return		0.006 (0.010)		-0.005 (0.019)
3-period-lagged house price return		0.002 (0.011)		0.004 (0.018)
4-period-lagged house price return		--		0.000 (0.012)
lagged inflation rate	0.034** (0.009)	0.026** (0.011)	0.039 (0.020)	0.053** (0.022)
composite loan-to-value ratio	-0.002 (0.215)	0.045 (0.173)	0.038 (0.219)	0.552 (0.465)
Change of composite loan-to-value ratio	0.001 (0.226)	0.047 (0.159)	0.009 (0.223)	0.109 (0.536)
change in income	0.372 (0.565)	-0.074 (0.481)	0.933 (0.793)	0.873 (0.807)
change of 3-month Treasury bill rate	0.004 (0.007)	0.006 (0.007)	-0.015 (0.019)	-0.027 (0.032)

Note: *20%, **10% indicate the corresponding significance levels. The numbers in parentheses refer to the standard errors of the coefficients.

I.5.4. Extended SEMs

We extend the SEMs in Section I.5.1 by adding cumulative fundamental-actual differences term, in order to identify the impacts of deviations from fundamental-driven values.

Assume that in each time period t the house price return, the mortgage rate and the default rate have their fundamental values determined by economic conditions.

$$
\begin{aligned}
HR_t^* &= f_1\left(\sum_{s=0}^{p_2} MR_{t-s}, \sum_{s=0}^{p_3} D_{t-s}, X\right) \\
MR_t^* &= f_2\left(\sum_{s=0}^{p_1} HR_{t-s}, \sum_{s=0}^{p_3} D_{t-s}, Y\right) \\
D_t^* &= f_3\left(\sum_{s=0}^{p_1} HR_{t-s}, \sum_{s=0}^{p_2} MR_{t-s}, Z\right)
\end{aligned}
\qquad \text{(I.3)}
$$

where HR_t^*, MR_t^* and D_t^* represent the fundamental values determined by equation (I.3). So we have

$$
\begin{aligned}
HR_t &= HR_t^* + \upsilon_t^1 \\
MR_t &= MR_t^* + \upsilon_t^2 \\
D_t &= D_t^* + \upsilon_t^3
\end{aligned}
$$

where υ_t^1, υ_t^2 and υ_t^3 are the error terms. Following Abraham and Hendershott (1996), the error terms could be described as an adjustment dynamics:

$$
\begin{aligned}
\upsilon_t^1 &= a_0 + \sum_{s=1}^{p_1} a_s HR_{t-s} + \alpha \sum_{i=1}^{t-1}\left(HR_i^* - HR_i\right) + \eta_t^1 \\
\upsilon_t^2 &= b_0 + \sum_{s=1}^{p_2} b_s MR_{t-s} + \beta \sum_{i=1}^{t-1}\left(MR_i^* - MR_i\right) + \eta_t^2 \\
\upsilon_t^3 &= c_0 + \sum_{s=1}^{p_3} c_s D_{t-s} + \gamma \sum_{i=1}^{t-1}\left(D_i^* - D_i\right) + \eta_t^3
\end{aligned}
$$

where $\sum_{i=1}^{t-1}\left(HR_i^* - HR_i\right)$, $\sum_{i=1}^{t-1}\left(MR_i^* - MR_i\right)$ and $\sum_{i=1}^{t-1}\left(D_i^* - D_i\right)$ are the cumulative fundamental-actual differences. η_t^1, η_t^2 and η_t^3 are the error terms. Putting these equations together, we get a simulataneous equations model:

$$\begin{aligned}
HR_t &= f_1\left(\sum_{s=0}^{p_2} MR_{t-s}, \sum_{s=0}^{p_3} D_{t-s}, X\right) + a_0 + \sum_{s=1}^{p_1} a_s HR_{t-s} + \alpha\sum_{i=1}^{t-1}\left(HR_i^* - HR_i\right) + \delta_1, \\
MR_t &= f_2\left(\sum_{s=0}^{p_1} HR_{t-s}, \sum_{s=0}^{p_3} D_{t-s}, Y\right) + b_0 + \sum_{s=1}^{p_2} b_s MR_{t-s} + \beta\sum_{i=1}^{t-1}\left(MR_i^* - MR_i\right) + \delta_2, \quad \text{(I.4)} \\
D_t &= f_3\left(\sum_{s=0}^{p_1} HR_{t-s}, \sum_{s=0}^{p_2} MR_{t-s}, Z\right) + c_0 + \sum_{s=1}^{p_3} c_s D_{t-s} + \gamma\sum_{i=1}^{t-1}\left(D_i^* - D_i\right) + \delta_3,
\end{aligned}$$

For such a simultaneous equations model, the typical estimation method is three-stage-least-square (3SLS). And another difficulty is that HR_t^*, MR_t^* and D_t^* themselves depend on the estimation results from the model. We follow the estimation method of Abraham and Hendershott (1996). The steps are as follows:

(1) estimating equation (I.4) without the cumulative fundamental-actual difference terms;

(2) calculating HR_i^*, MR_i^* and D_i^* *(i=1,2,...,t-1),* so that we can obtain the cumulative fundamental-actual difference terms $\sum_{i=1}^{t-1}\left(HR_i^* - HR_i\right)$, $\sum_{i=1}^{t-1}\left(MR_i^* - MR_i\right)$ and $\sum_{i=1}^{t-1}\left(D_i^* - D_i\right)$.

(3) Adding $\sum_{i=1}^{t-1}\left(HR_i^* - HR_i\right)$, $\sum_{i=1}^{t-1}\left(MR_i^* - MR_i\right)$ and $\sum_{i=1}^{t-1}\left(D_i^* - D_i\right)$ and re-estimating equation (I.4).

(4) Recalculating HR_i^* , MR_i^* , D_i^* *(i=1,2,...,t-1)* and $\sum_{i=1}^{t-1}\left(HR_i^* - HR_i\right)$, $\sum_{i=1}^{t-1}\left(MR_i^* - MR_i\right)$, $\sum_{i=1}^{t-1}\left(D_i^* - D_i\right)$.

(5) Re-estimating equation (I.4) till the coefficients converge.

This method is based on the idea that if the fundamental-driven terms, the lag term and the cumulative fundamental-actual difference term are uncorrelated, the coefficient estimation would be stable.

I.5.5. Extended SEMs: Estimation Results

Table I.4 exhibits part of the three-stage-least-square regression results for the extended SEM with the FHFA's index[19].

The fundamental-actual difference term presents the cumulative effects of the fundamental-driven factors on the actual data. Since all the serial correlation coefficients in our model are positive, a positive coefficient on the fundamental-actual difference term displays the deviation from the lagged actual value, while a negative coefficient strengthens the lagged actual value.

Basically, the actual house price returns converge around 3-4 percent each quarter of the difference. Under most cases, the difference term in house price equation is statistically significant.

The actual default rate deviates 0.4-1 percent and, in three-equation models, the actual mortgage rate converges around 0.5 percent, although highly statistically significant.

[19] We do not report the estimation results of the extended SEM with Case-Shiller's index. Those results can be obtained by request.

Table I.4: Part of 3SLS Regression Results for Model 3 with the FHFA's house price returns

Three-Equation SEM with 1-period lag terms			
House Price Return Equation			
Variable	Regression 1	Regression 2	Regression 3
1-period-lagged house price return	0.563** (0.084)	0.564** (0.103)	0.611** (0.085)
Lagged Deviation Term of House Price	0.038** (0.019)	0.037** (0.021)	0.048** (0.017)
Default Rate Equation			
1-period-lagged default rate	1.043** (0.035)	1.059** (0.036)	1.079** (0.037)
Lagged Deviation Term of Default Rate	--	0.004 (0.004)	0.009** (0.004)
Mortgage Rate Equation			
1-period-lagged 30-year fixed mortgage rate	0.949** (0.017)	0.962** (0.016)	0.959** (0.020)
Lagged Deviation Term of Mortgage Rate	--	--	0.005** (0.001)
Three-Equation SEM with multi-period lag/change terms			
House Price Return Equation			
Variable	Regression 1	Regression 2	Regression 3
1-period-lagged house price return	0.493** (0.113)	0.468** (0.114)	0.512** (0.105)
2-period-lagged house price return	0.031 (0.131)	0.036 (0.131)	0.006 (0.121)
3-period-lagged house price return	0.270** (0.140)	0.327** (0.123)	0.284** (0.109)
Lagged Deviation Term of House Price	0.014 (0.018)	0.041** (0.016)	0.031** (0.012)
Default Rate Equation			
1-period-lagged default rate	1.013** (.038)	1.055** (0.060)	1.056** (0.065)
1-period-lagged change of default rate	0.231** (0.107)	0.156 (0.189)	0.090 (0.196)
2-period-lagged change of default rate	-0.018 (0.108)	-0.075 (0.160)	-0.106 (0.178)
3-period-lagged change of default rate	0.267** (0.122)	0.202 (0.248)	0.035 (0.240)
Lagged Deviation Term of Default Rate	--	0.006 (0.009)	0.010 (0.008)
Mortgage Rate Equation			
1-period-lagged 30-year fixed mortgage rate	0.934* (0.019)	0.956** (0.016)	0.958** (0.017)
1-period-lagged change of 30-year fixed mortgage rate	0.009 (0.103)	0.097 (0.082)	0.062 (0.098)
2-period-lagged change of 30-year fixed mortgage rate	0.061 (0.104)	-0.007 (0.083)	-0.031 (0.092)
3-period-lagged change of 30-year fixed mortgage rate	-0.019 (0.085)	-0.040 (0.067)	-0.065 (0.078)
Lagged Deviation Term of Mortgage Rate	--	--	0.003** (0.002)

Note: **10% indicate the corresponding significance levels. The numbers in parentheses refers to the standard errors of the coefficients

Regression 1 only includes the difference term in house price return equation. Regression 2 contains the difference terms in house price return equation and the default rate equation. Regression 3 has the difference terms in all the three equations.

I.6. Model Predictions

In this part, we first try to make some prediction performance comparisons among different models. Second, we use the data up to the fourth quarter of 2007, in order to compare the predicted values with the actual data for the first two quarters of 2008. Third, we use the data up to the second quarter of 2008 to make more predictions.

I.6.1. Prediction Performance Comparison

Generally, different models are specified for different purposes and are used to predict over various horizons. We do not want to jump to a conclusion as to which model is better. We conduct the comparisons, based on the quarterly predictions of the AR model, the VAR model, the one-period lagged SEM and the multi-period lagged SEM, for the two periods: (1) from first quarter 2005 through second quarter 2008; (2) from the first quarter of 2007, the beginning period of the mortgage crisis, through second quarter 2008.

For the period from first quarter 2005 through second quarter 2008, we compare eight-quarter-ahead predictions by a rolling window analysis. For example, we first employ the data through fourth quarter 2004 to estimate, and make predictions for first quarter 2005 through fourth quarter 2006, based on the estimates. Then we estimate via the data through first quarter 2005 and predict for second quarter 2005 through first quarter 2007. Since the data used in this paper are only through second quarter 2008, when observations are not available for the latest or future periods, the prediction results are dropped from the comparison sample. The predictions for SEM's are conditional predictions, so we use the actual observations for exogenous variables.

For the period from first quarter 2007 through second quarter 2008, we compare four-quarter-ahead predictions by a rolling window analysis.

The measure of prediction accuracy is the root mean squared error (RMSE). Then for *s*-quarter-ahead prediction performance, the formula is

$$RMSE = \left[\frac{1}{T} \sum_{t=1}^{T} \left(Y_t - {}_s\hat{Y}_t \right) \right]^{1/2},$$

where Y_t is the actual observation at time t for house price return, mortgage interest rate or default rate, ${}_s\hat{Y}_t$ is the prediction made *s* quarters earlier, and T is the total numbers of the predictions made *s* quarters earlier during the above specified period.

Table I.5 exhibits the prediction comparison results using the FHFA's or Case-Shiller's house price returns, measured by RMSE, for the period from first quarter 2005 through second quarter 2008. Table I.6 exhibits the prediction comparison results for the period from first quarter 2007 through second quarter 2008.

We have to admit that the samples we explored during the test periods are small, especially in the second period, which may somewhat affect the results. However, a few results are still clear. In forecasting house price returns and default rates, the VAR model and SEM models produce better predictions results than the AR model. These results are also consistent with the Granger-causality test. One important factor which affects the results may be that our comparison periods are near or in the subprime crisis, when the increased mortgage default rates may have heavier impacts on house prices.

Table I.5: Prediction Performance Comparison, Root Mean Squared Error, 2005Q1—2008Q2

The left side of this table exhibits the results using the FHFA's house price returns; the right side of the table displays the results using the Case-Shiller's house price returns.

	the FHFA's House Price Return				the Case-Shiller's House Price Return			
Prediction Horizon	AR	VAR	one-period lagged SEM	multi-period lagged SEM	AR	VAR	one-period lagged SEM	multi-period lagged SEM
1	0.899	0.757	0.758	0.776	1.304	1.065	1.168	1.221
2	1.177	1.065	0.98	0.859	2.039	1.685	2.196	2.327
3	1.268	1.194	1.004	0.858	2.411	2.091	2.597	2.499
4	1.302	1.393	1.044	1.212	3.074	2.837	3.578	3.438
5	1.506	1.692	0.966	1.34	3.574	3.400	4.201	4.011
6	1.648	1.896	0.976	1.481	4.811	4.145	5.133	4.529
7	1.771	2.104	1.012	1.78	5.356	4.924	6.142	5.323
8	1.875	2.266	1.092	1.967	5.862	5.839	7.506	6.324
	Mortgage Interest Rate				Mortgage Interest Rate			
Prediction Horizon	AR	VAR	one-period lagged SEM	multi-period lagged SEM	AR	VAR	one-period lagged SEM	multi-period lagged SEM
1	0.251	0.313	0.127	0.273	0.251	0.408	0.129	0.133
2	0.438	0.521	0.144	0.32	0.438	0.845	0.197	0.194
3	0.533	0.507	0.262	0.346	0.533	1.032	0.315	0.267
4	0.635	0.53	0.461	0.451	0.635	1.312	0.419	0.360
5	0.735	0.593	0.666	0.578	0.735	1.570	0.452	0.423
6	0.885	0.589	0.752	0.646	0.885	1.722	0.535	0.499
7	0.961	0.391	0.827	0.616	0.961	1.875	0.638	0.532
8	1.004	0.341	0.898	0.705	1.004	2.040	0.825	0.567
	Mortgage Default Rate				Mortgage Default Rate			
Prediction Horizon	AR	VAR	one-period lagged SEM	multi-period lagged SEM	AR	VAR	one-period lagged SEM	multi-period lagged SEM
1	0.103	0.103	0.106	0.098	0.103	0.111	0.079	0.090
2	0.194	0.194	0.201	0.184	0.194	0.168	0.137	0.159
3	0.283	0.285	0.317	0.278	0.283	0.239	0.203	0.237
4	0.396	0.402	0.426	0.389	0.396	0.345	0.279	0.339
5	0.469	0.481	0.501	0.461	0.469	0.403	0.353	0.434
6	0.533	0.55	0.537	0.49	0.533	0.455	0.386	0.488
7	0.574	0.591	0.574	0.504	0.574	0.462	0.397	0.509
8	0.605	0.617	0.618	0.521	0.605	0.406	0.368	0.527

Table I. 6: Prediction Performance Comparison, Root Mean Squared Error, 2007Q1—2008Q2

The left side of this table exhibits the results using the FHFA's house price returns; the right side of the table displays the results using the Case-Shiller's house price returns.

	the FHFA's House Price Return				the Case-Shiller's House Price Return			
Prediction Horizon	AR	VAR	one-period lagged SEM	multi-period lagged SEM	AR	VAR	one-period lagged SEM	multi-period lagged SEM
1	1.418	0.852	0.765	0.623	2.950	0.722	0.487	0.741
2	2.589	1.822	1.181	0.506	8.109	0.999	5.710	6.971
3	3.39	2.585	1.563	0.455	7.269	0.762	5.259	5.662
4	4.102	3.752	2.301	1.109	3.732	1.120	15.716	16.767
	Mortgage Interest Rate				Mortgage Interest Rate			
Prediction Horizon	AR	VAR	one-period lagged SEM	multi-period lagged SEM	AR	VAR	one-period lagged SEM	multi-period lagged SEM
1	0.062	0.148	0.027	0.075	0.062	0.192	0.031	0.013
2	0.221	0.549	0.048	0.129	0.221	0.952	0.086	0.050
3	0.201	0.593	0.137	0.192	0.201	0.578	0.217	0.104
4	0.063	0.644	0.536	0.443	0.063	0.224	0.377	0.134
	Mortgage Default Rate				Mortgage Default Rate			
Prediction Horizon	AR	VAR	one-period lagged SEM	multi-period lagged SEM	AR	VAR	one-period lagged SEM	multi-period lagged SEM
1	0.017	0.016	0.019	0.015	0.017	0.021	0.011	0.014
2	0.073	0.07	0.087	0.068	0.073	0.05	0.046	0.057
3	0.182	0.181	0.267	0.195	0.182	0.121	0.143	0.141
4	0.42	0.424	0.553	0.458	0.42	0.312	0.33	0.361

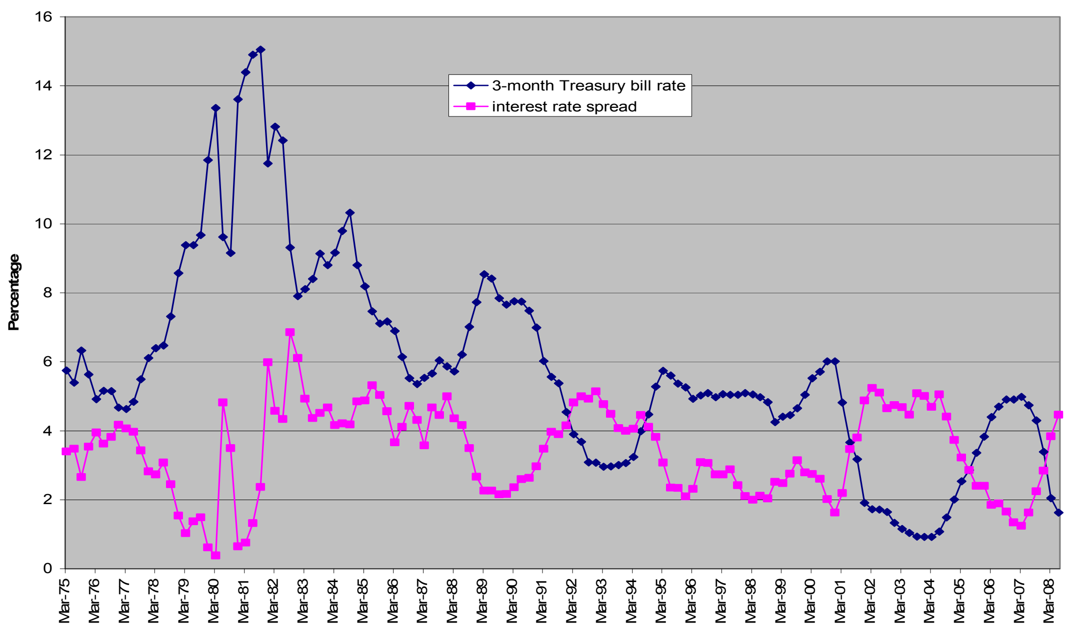
I.6.2. Testing of Predictive Performance using data through the end of 2007

Prediction via VAR model

We first examine relationships between the 3-month Treasury bill rates, the 30-year fixed mortgage rate, and mortgage rate spreads (as the differences between 30-year fixed mortgage rates and 3-month Treasury bill rates). Although the 3-month Treasury bill rates can come down to near zero during some periods of Fed easing, the mortgage rate spreads tend to move upward during these economic periods (See Figure I.5). This opposite movement of mortgage spreads will keep 30-year fixed mortgage rates above a certain level. In fact, the historical 30-year mortgage rates have never come below 5%. Accordingly, in our econometric modeling of future mortgage rates, we put a constraint on future 30-year fixed mortgage rates and they will always be no less than 4 percent.

Figure I.5: Comparison between 3-month Treasury bill rates and mortgage spreads

Mortgage spreads are the differences between 30-year fixed mortgage rates and the 3-month Treasury bill. Although the 3-month Treasury bill rates can come down to near zero during some time periods of Fed easing, the mortgage rate spreads tend to move upward during these economic periods, keeping 30-year fixed mortgage rates above a certain level .



For both house price indices, we make 3-year forecasts via VAR, using historical data up to the fourth quarter of 2007. By comparison, we also make forecasts via Auto-Regressive models (AR). We employ Monte-Carlo simulations to estimate confidence intervals for the predictions.

The actual quarter-over-quarter the FHFA's house price returns for the first two quarters in 2008 have continuously deteriorated. The return is -0.23% in the first quarter and -1.45% in the second quarter of 2008, which is the worst quarter-over-quarter return since 1975. Figure I.6 displays the predicted values of the FHFA's house price returns for the next three years from first quarter 2008 through fourth quarter 2010. The mean values of predicted house price returns per the AR model are always positive over time since 2008, which deviates from the actual data. On the contrary, the mean values of predicted house price returns per VAR models are mainly negative. The predictions based on VAR reach the lowest point in third quarter 2009. Additionally, although the confidence intervals per both AR and VAR fail to exactly catch the huge deterioration in the second quarter of 2008, the 90% confidence limit from VAR is relatively close to the actual data.

The actual quarter-over-quarter Case-Shiller's house price returns for the first two quarters in 2008 show a different trend. The return is -6.79% in the first quarter, which is lowest since 1987, and -4.29% in second quarter 2008, which is better than the previous one. Figure I.7 displays the predicted values of the Case-Shiller's house price returns for the next three years from first quarter 2008 through fourth quarter 2010. The 90% confidence limits catch the huge deterioration in first quarter 2008. The mean values of predicted Case-Shiller's house price returns per the VAR model are recovering a little bit quicker than the ones per the AR model.

The actual national default rates for the first two quarters in 2008 have deteriorated further, with 1.63% in first quarter 2008 and 1.83% in second quarter 2008 the highest since 1979. Figure I.8 shows the expected predictions of national default rates by the AR model, the VAR model with the FHFA's house price returns, and the VAR model with the Case-Shiller's house price returns. The three models roughly catch the trends during the first two quarters of 2008. While the default rates per the AR model or the VAR model with the FHFA's house price returns display a continuously increasing trend, the expected predictions per the VAR model with the Case-Shiller's house price returns reach the highest in 2009.

When investigating the predicted mortgage rate, the results from the three models diverge, displaying complicated relationships with the other variables.

These above results clearly show the impacts of mortgage default on the housing market, no matter which house price index we use.

Figure I.6: Actual vs Predicted the FHFA's House Price Returns

The house price returns are from the FHFA's index. The models use the data through the end of 2007 and there are 3-year predictions through fourth quarter 2010.

Figure I.6a: Predictions based on AR model.

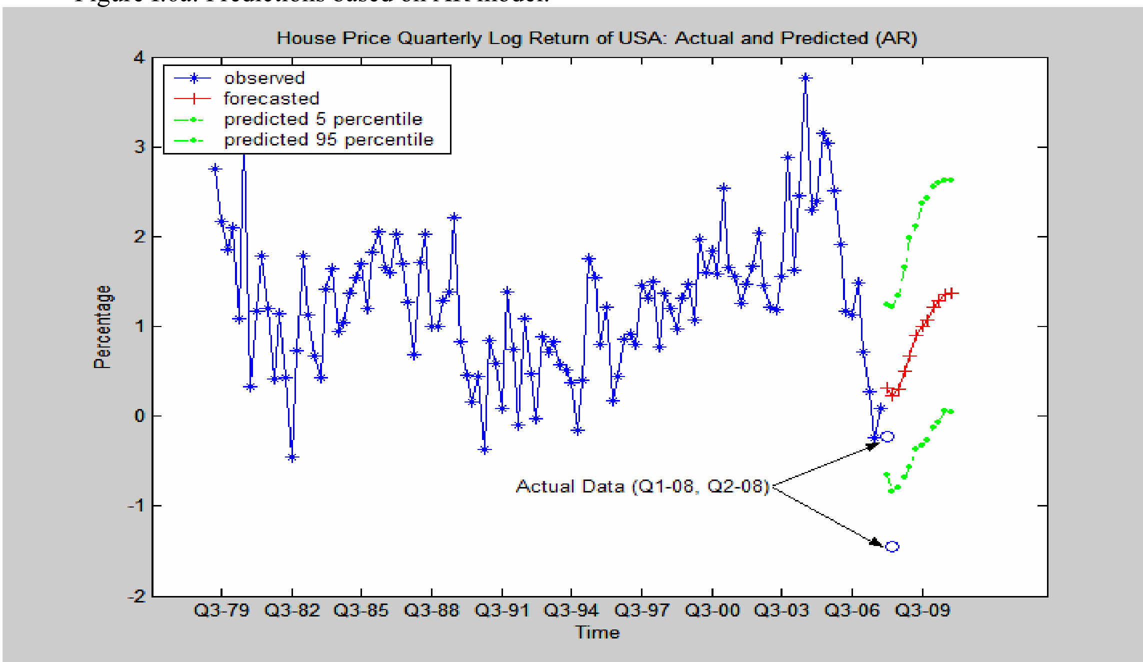


Figure I.6b: Predictions based on VAR model.

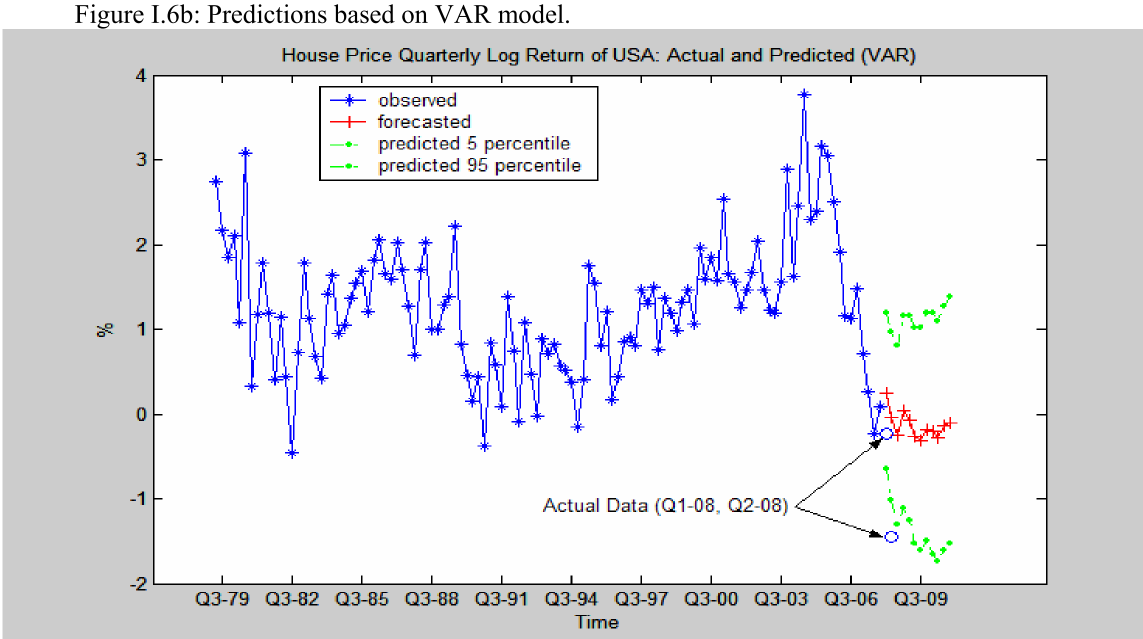


Figure I.7: Actual vs Predicted Case-Shiller's House Price Returns

The house price returns are from the Case-Shiller's index. The models use the data through the end of 2007 and there are 3-year predictions through fourth quarter 2010.

Figure I.7a: Predictions based on AR model.

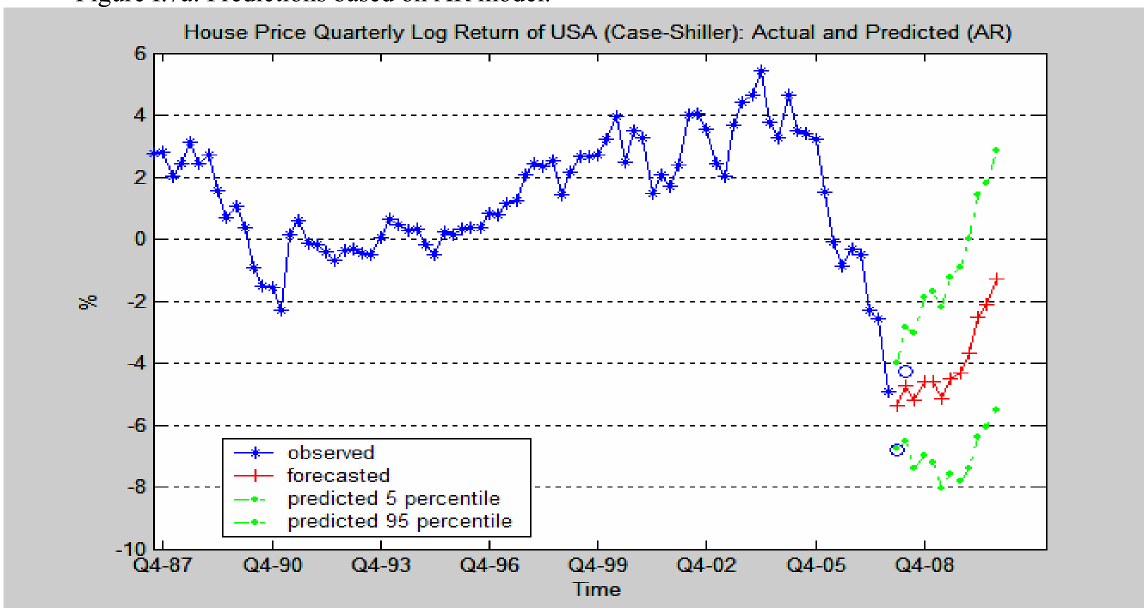


Figure I.7b: Predictions based on VAR model.

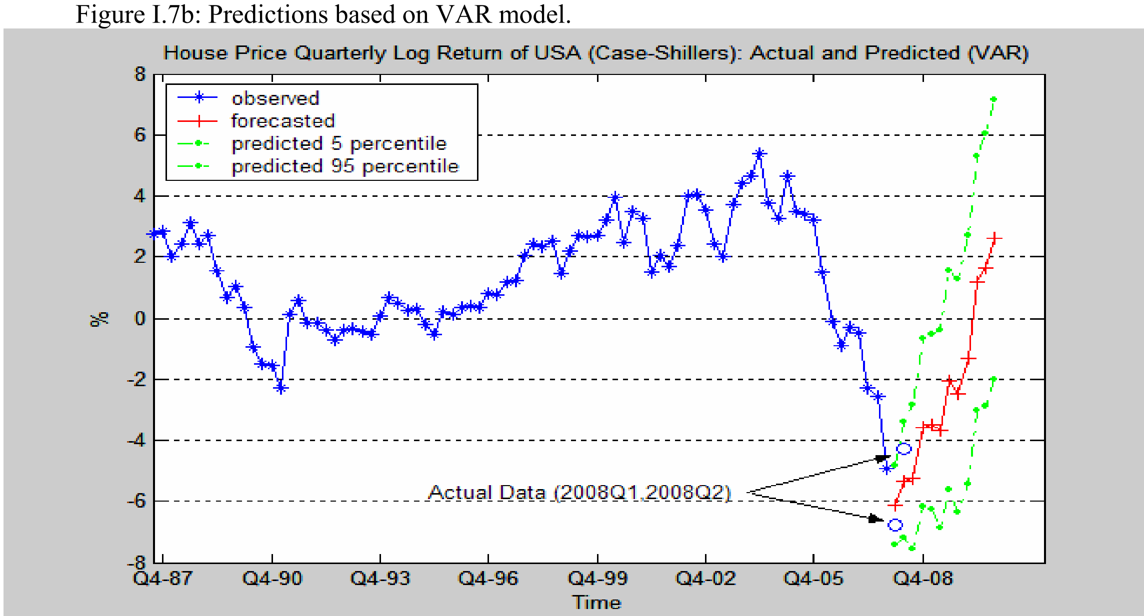


Figure I.8: Actual vs Predicted National Default Rates

The models use the data through the end of 2007 and there are 3-year predictions through fourth quarter 2010.

Figure I.8a: Predictions based on AR model

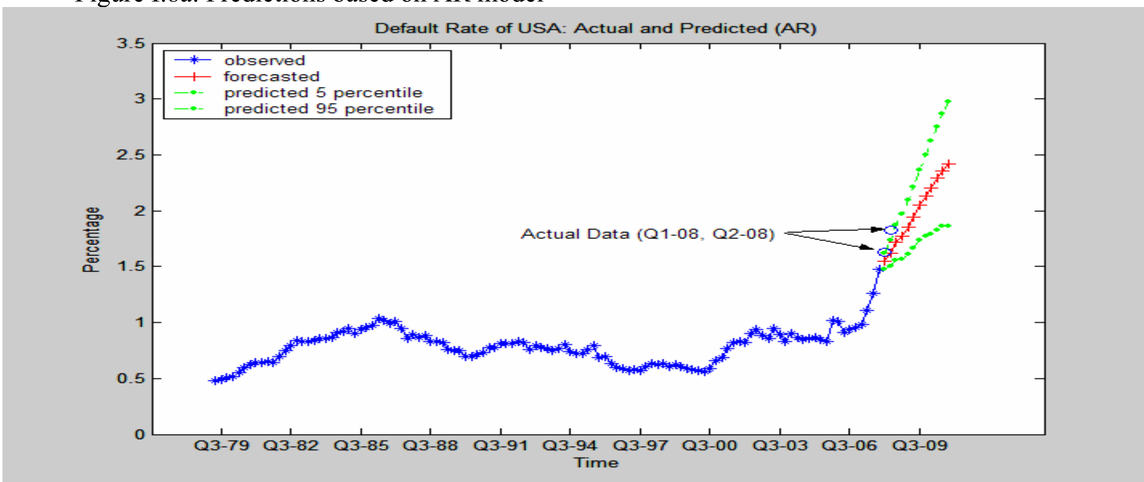


Figure I.8b: Predictions based on VAR model with the FHFA's House Price Returns

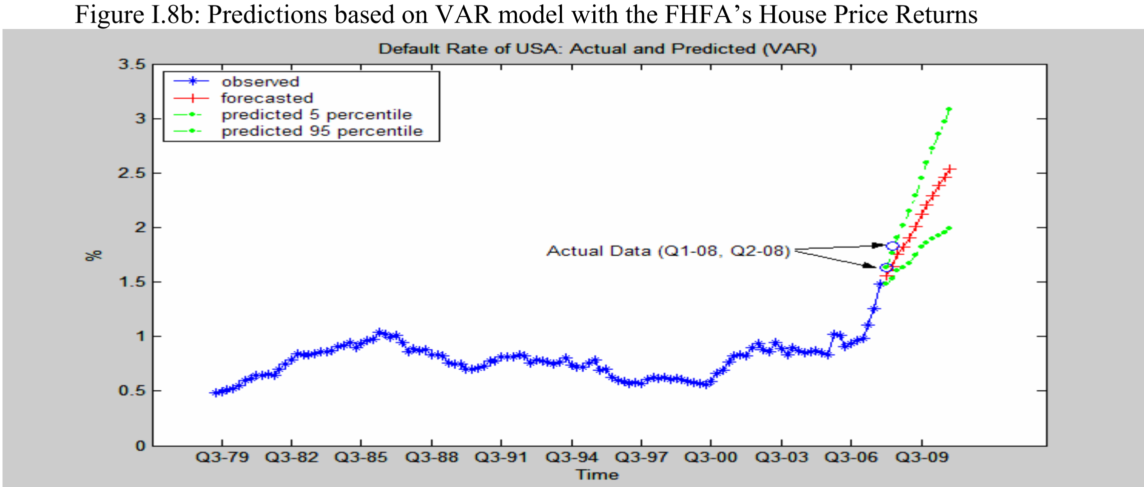


Figure I.8c: Predictions based on VAR model with the Case-Shiller's House Price Returns

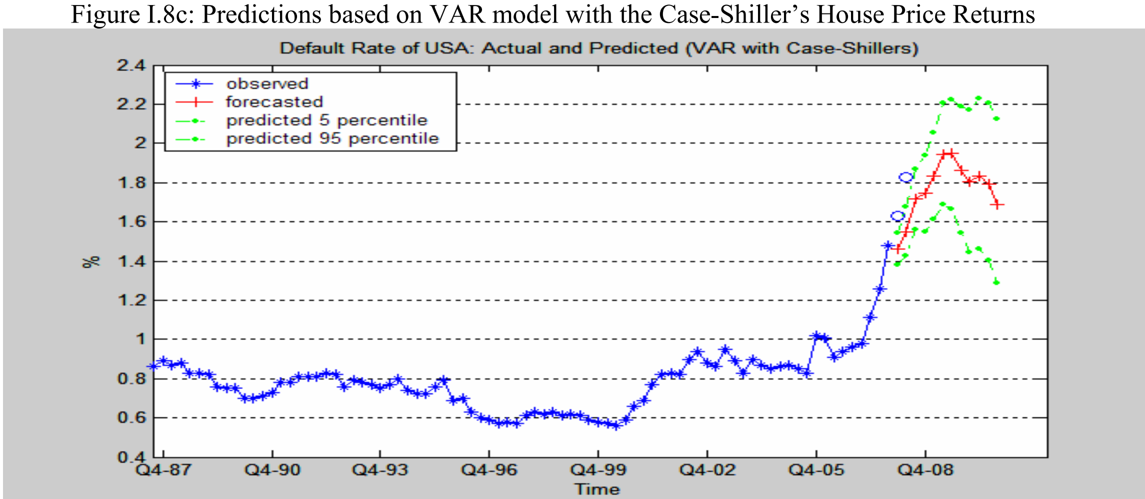


Figure I.9: Actual vs. Predicted National Mortgage Rates

The models use the data through the end of 2007 and there are 3-year predictions through fourth quarter 2010.

Figure I.9a: Predictions based on AR model

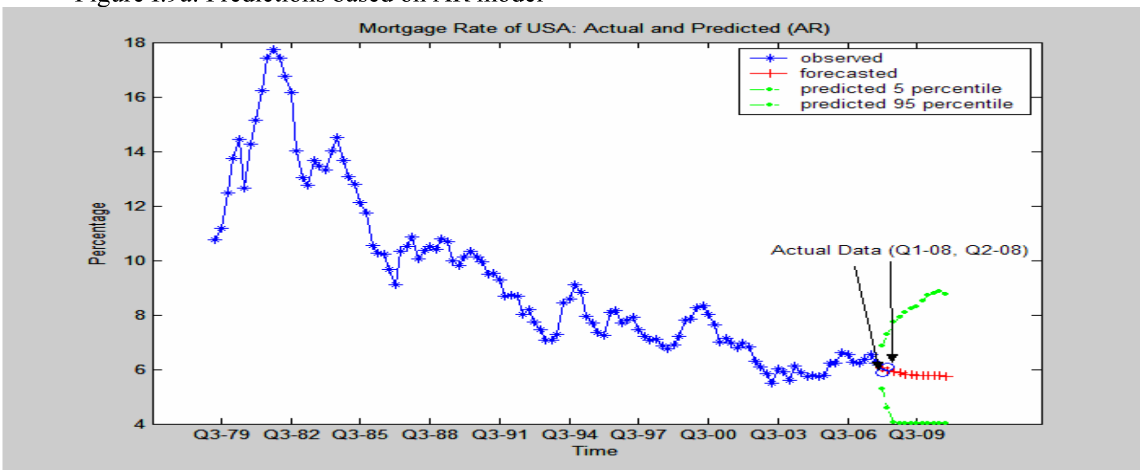


Figure I.9b: Predictions based on VAR model with the FHFA's House Price Returns

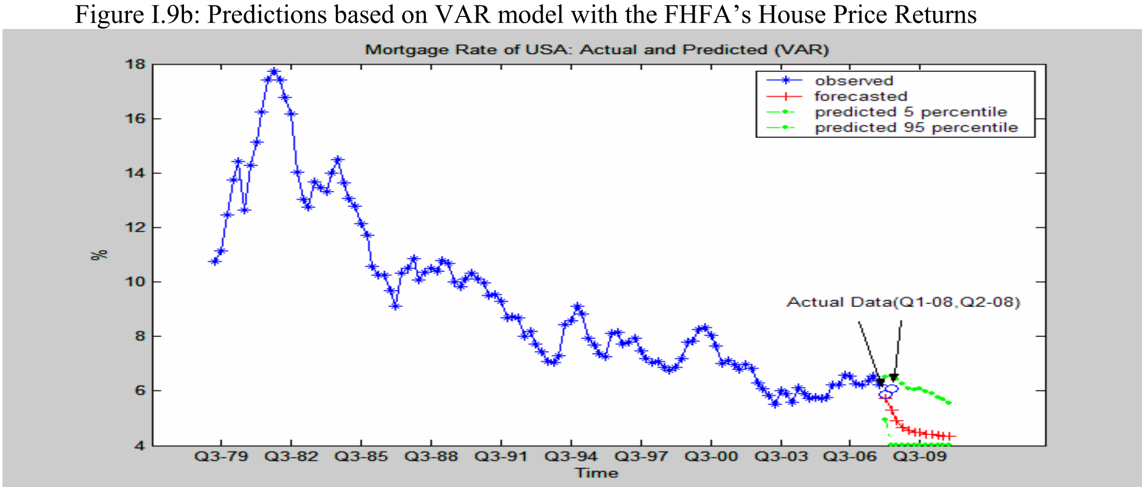
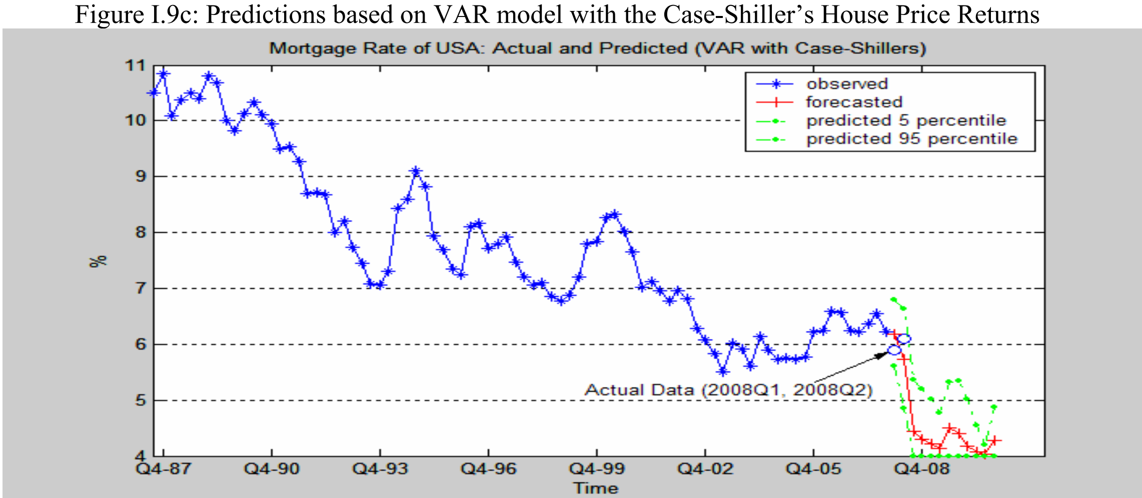


Figure I.9c: Predictions based on VAR model with the Case-Shiller's House Price Returns



Prediction per SEM

Based on the known values of the exogenous variables in the first two quarters of 2008, we calculate conditional predictions[20] per SEM. The expected results per the model with the FHFA's house price returns and the Case-Shiller's house price returns are shown in Table I.7 and I.8 respectively.

The multi-period-lagged SEM obtains the predicted the FHFA house price returns with the means of -0.24% and -0.83% (-0.89% if predicted dynamically) and with the confidence intervals of [-1.31%, 0.81] and [-1.91%, 0.29%] ([-2.07% 0.33%] if predicted dynamically) in the first two quarters of 2008. Similarly, for default rates and mortgage rates, the predicted means from the multi-period-lagged SEM are quite close to the actual values.

For the model with the Case-Shiller's house price returns, the main exception is the predicted result for the Case-Shiller's house price return in the second quarter of 2008, which deviates substantially from the actual value.

[20] The unconditional predictions do not show much improvement, compared with the results from VAR. We do not present the results here.

Table 7: Conditional Predictions of the FHFA's House Price Returns, Mortgage Rates and Default Rates

the FHFA's House Price Returns (%)											
	actual	VAR		1-period lag SEM				multi-period lag SEM			
			90% Conf Interval	one step forecast	90% Conf Interval	dynamic forecast	90% Conf Interval	one step forecast	90% Conf Interval	dynamic forecast	90% Conf Interval
3/31/2008	-0.23	0.25	[-0.65 1.19]	0.24	[-0.79 1.34]			-0.24	[-1.31 0.81]		
6/30/2008	-1.45	-0.04	[-1.02 0.96]	0.04	[-1.01 1.10]	0.27	[-0.87 1.43]	-0.83	[-1.91 0.29]	-0.89	[-2.07 0.33]
Default Rate (%)											
	actual	VAR		1-period lag SEM				multi-period lag SEM			
			90% Conf Interval	one step forecast	90% Conf Interval	dynamic forecast	90% Conf Interval	one step forecast	90% Conf Interval	dynamic forecast	90% Conf Interval
3/31/2008	1.63	1.56	[1.48 1.63]	1.59	[1.51 1.67]			1.66	[1.57 1.74]		
6/30/2008	1.83	1.64	[1.52 1.76]	1.73	[1.65 1.81]	1.68	[1.56 1.81]	1.79	[1.71 1.87]	1.82	[1.69 1.94]
Mortgage Rate (%)											
	actual	VAR		1-period lag SEM				multi-period lag SEM			
			90% Conf Interval	one step forecast	90% Conf Interval	dynamic forecast	90% Conf Interval	one step forecast	90% Conf Interval	dynamic forecast	90% Conf Interval
3/31/2008	5.88	5.76	[5.01 6.53]	6.00	[5.45 6.57]			5.96	[5.33 6.57]		
6/30/2008	6.09	5.33	[4.00 6.68]	6.11	[5.52 6.67]	6.28	[5.49 7.10]	6.12	[5.49 6.77]	6.19	[5.10 7.35]

Table 8: Conditional Predictions of the Case-Shiller's House Price Returns, Mortgage Rates and Default Rates

the Case-Shiller's House Price Returns (%)											
	actual	VAR		1-period lag SEM				multi-period lag SEM			
			90% Conf Interval	one step forecast	90% Conf Interval	dynamic forecast	90% Conf Interval	one step forecast	90% Conf Interval	dynamic forecast	90% Conf Interval
3/31/2008	-6.79	-6.14	[-7.74 -4.55]	-6.21	[-8.74 -3.65]			-6.46	[-8.30 -4.63]		
6/30/2008	-4.30	-5.36	[-7.65 -3.08]	-7.28	[-9.73 -4.69]	-6.97	[-9.97 -3.81]	-7.00	[-8.94 -5.06]	-6.62	[-9.11 -4.18]
Default Rate (%)											
	actual	VAR		1-period lag SEM				multi-period lag SEM			
			90% Conf Interval	one step forecast	90% Conf Interval	dynamic forecast	90% Conf Interval	one step forecast	90% Conf Interval	dynamic forecast	90% Conf Interval
3/31/2008	1.63	1.61	[1.52 1.69]	1.68	[1.60 1.77]			1.62	[1.52 1.73]		
6/30/2008	1.83	1.86	[1.74 1.98]	1.82	[1.74 1.91]	1.87	[1.76 1.98]	1.82	[1.71 1.93]	1.81	[1.66 1.95]
Mortgage Rate (%)											
	actual	VAR		1-period lag SEM				multi-period lag SEM			
			90% Conf Interval	one step forecast	90% Conf Interval	dynamic forecast	90% Conf Interval	one step forecast	90% Conf Interval	dynamic forecast	90% Conf Interval
3/31/2008	5.88	5.81	[5.30 6.31]	5.74	[5.50 5.99]			5.95	[5.65 6.25]		
6/30/2008	6.09	4.98	[4.27 5.71]	6.16	[5.92 6.41]	6.00	[5.67 6.35]	6.42	[6.12 6.73]	6.48	[5.99 6.92]

I.6.3. Model Predictions Using Data through 2008Q2

We re-estimate the VAR model using the data through second quarter 2008 and make predictions. And the prediction results are graphed in Figure I.10 with the FHFA's house price returns and in Figure I.11 with the Case-Shiller's house price returns.

The prediction results show great differences due to the different trends for the two indices. As we mentioned, the FHFA's house price returns reach the lowest value in second quarter 2008, while the Case-Shiller's house price returns have the lowest value in first quarter 2008 and are somewhat higher in second quarter 2008. Additionally, from the end of 2006 through second quarter 2008, the FHFA's house price index only decreased by about 1 percent, while the Case-Shiller's index dropped down by around 20 percent. The gaps between the two indices may come from the differences in their data sources and calculation methods, which is beyond our paper's scope.

On an expected value basis, the future level of the FHFA's house price returns will remain negative and reach the lowest value in 2010 and increase slowly thereafter, although it may take quite a few years for the house price returns to become positive. Based on the 90% confidence limits, if predicting optimistically, the house price returns may become positive again after 2010.

The expected Case-Shiller's house price returns tend to become positive after 2010. Figure 11 shows that default rates reach the highest value in 2010 and decrease slowly thereafter.

If we combine these two sets of results, we could say that, considering only the internal relationships among house price returns, mortgage rates and default rates, ignoring the effects of external factors, the year 2010 is an important turning point for house price returns and default rates.

Figure I.10: Predictions via VAR with the FHFA's House Price Returns

The models use the data through second quarter 2008 and there are 3-year predictions through second quarter 2011.

Figure I.10a: Predictions of the FHFA's house price returns

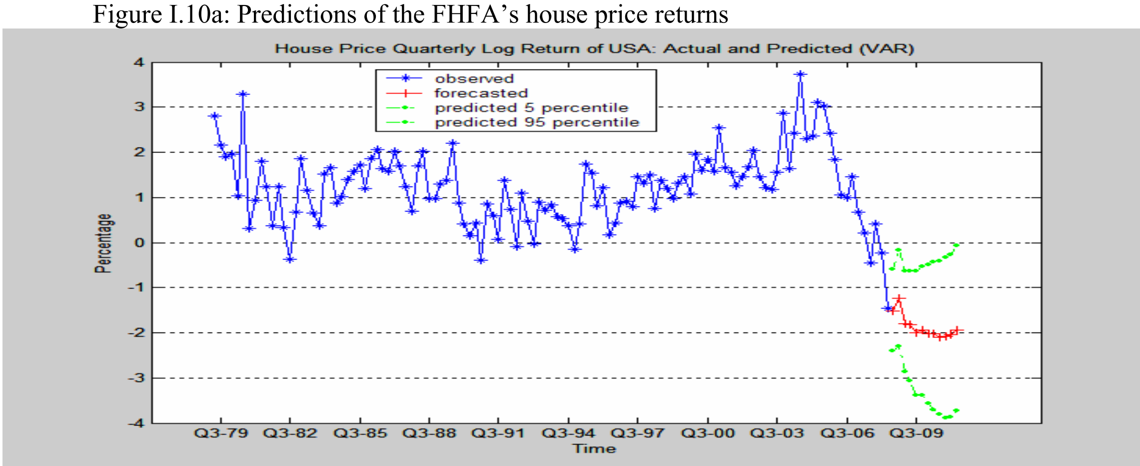


Figure I.10b: Predictions of Default Rate

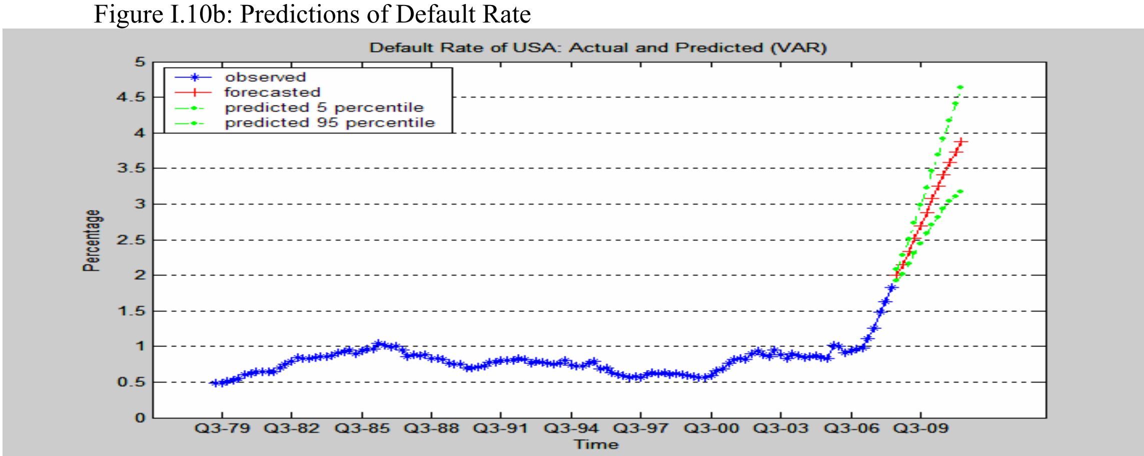


Figure I.10c: Predictions of Mortgage Rate

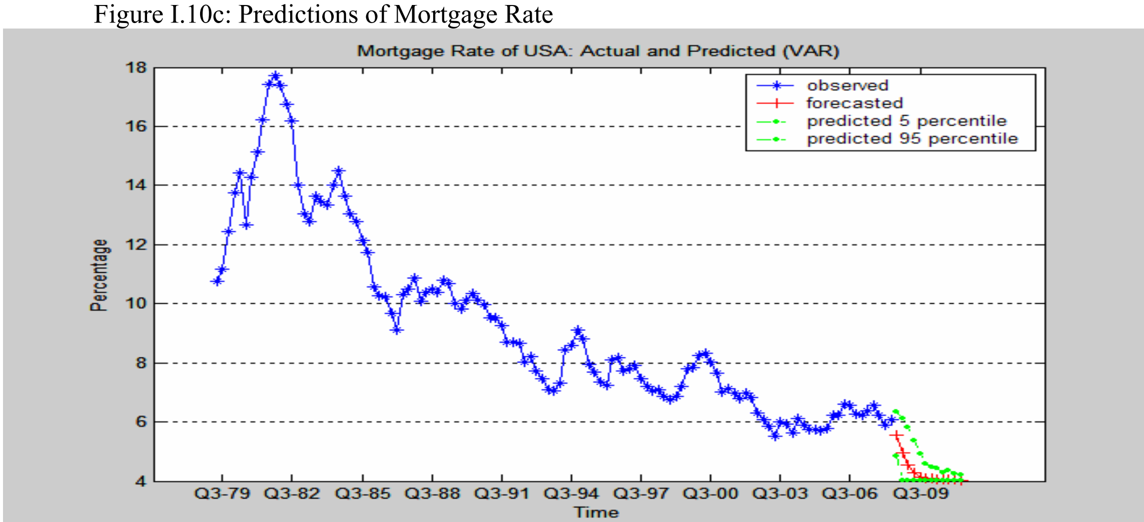


Figure I.11: Predictions via VAR with the Case-Shiller's House Price Returns

The model use data through second quarter 2008 and there are 3-year predictions through second quarter 2011.

Figure I.11a: Predictions of the Case-Shiller's house price returns

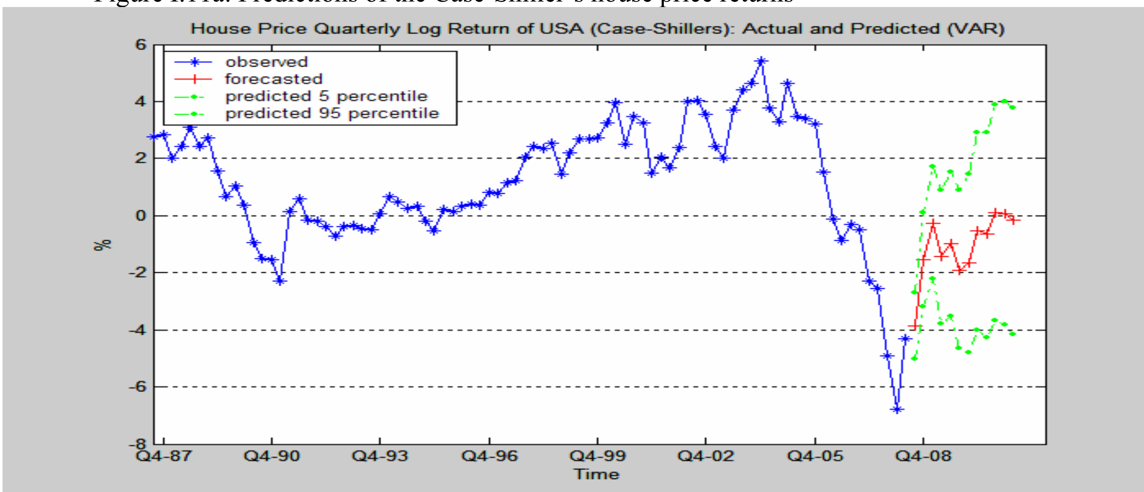


Figure I.11b: Predictions of Default Rate

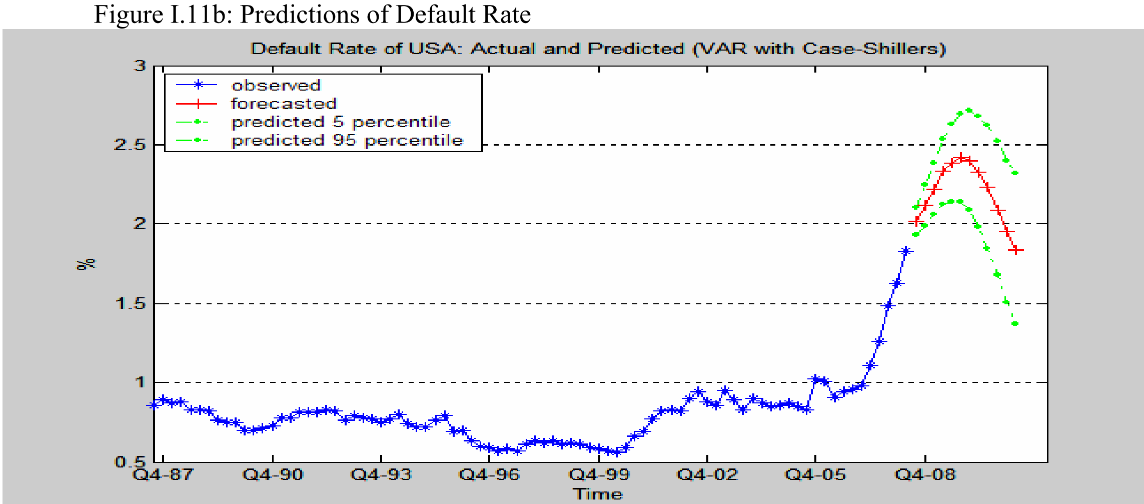
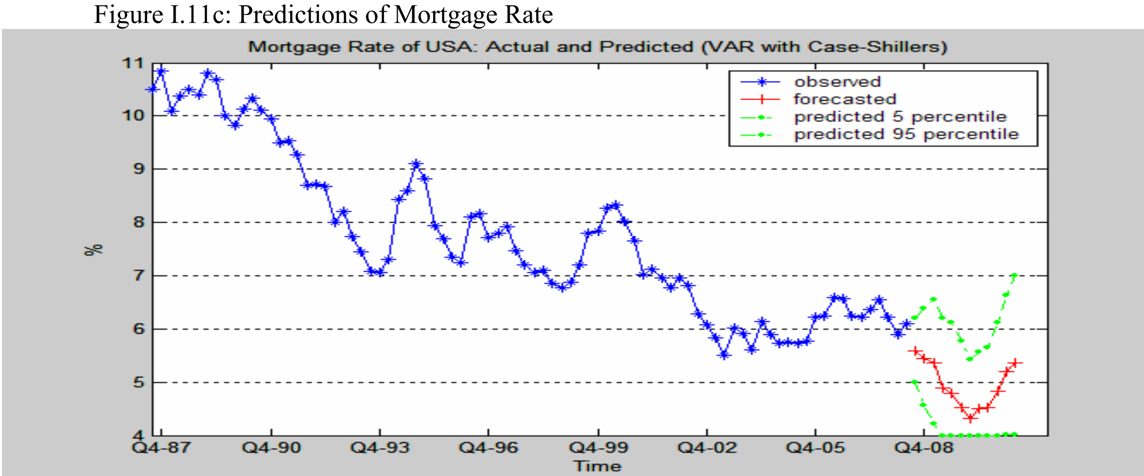


Figure I.11c: Predictions of Mortgage Rate



I.7. Conclusion

Based on the Granger-Causality test, we present VAR and SEM models to describe the dynamic relations of house price returns, mortgage rates and default rates. With their structural form, simultaneous equation models can explain the relationships more clearly. By investigating both the FHFA's and Case-Shiller's house price returns, we find the interactive negative relationship between house price returns and default rates. For example, holding all other factors constant, two consecutive 1% increases in default rates can drive the FHFA's house price returns down by about 7.64% and the Case-Shiller's current house price return down by about 18%. Conversely, two consecutive 1% decreases of the FHFA's or Case-Shiller's house price returns can push the current default rate up by 0.09 percent or 0.04 percent, respectively. The effects of mortgage rates show different results for models with the two different house price indices, reflecting complicated relationships.

In SEM models, the three level variables exhibit high serial correlations, reflecting strong momentum effects. In the extended SEM model, the house prices display a long-term adjustment process toward the fundamental values.

When making predictions using data through second quarter 2008, we observe that mortgage default rates have large impacts on house price returns, and vice versa. So the unfolding mortgage default experience can affect the recovery of the housing market. According to the VAR model, only considering the inter-relationships among house price returns, mortgage rates and default rates, the year 2010 will probably be an important turning point for both house price returns and mortgage default rates. We add caveats for interpreting these mechanical forecasts: they do not reflect many important dynamical factors that will strongly affect the housing markets, for instance, the various government mortgage modification programs, and the inventory of excess housing units.

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Chapter II Intrinsic Values and Dynamics of House Prices

Abstract

Following the methodology of co-integration, we first construct four succinct measures to display the possible intrinsic values of house prices. The fifth measure is defined as a weighted average of the first four. Estimated via the FHFA's House Price Index, all of the five measures exhibit overvalued house prices by the second quarter of 2008. On the contrary, using the S&P/Case-Shiller's Home Price Index, three measures show undervalued house prices by the second quarter of 2008.

In the short run, house price return dynamics are investigated by dynamic adjustments following Capozza et al (2002) and error correction models. The estimations reflect gradual adjustments towards the long-run intrinsic values. The impacts of the mortgage credit market on house price returns are also analyzed.

Furthermore, we examine the possible overshooting problem of house price returns. By both analytical derivations and simulations, we demonstrate the effects of the coefficients on overshooting.

II.1. Introduction

House prices are always an important indicator of an economy and have a strong impact on the economy. In particular, the falling house prices since late 2006 are regarded as one of the factors which trigger the mortgage crisis.

On the other side, house prices are determined by a broad array of economic variables. Existing academic literature has investigated a lot about the intrinsic values (or fundamental values) and bubbles of house prices.

Some papers use the dominant theory of asset pricing, which focuses on the paradigm that stock prices equal to the sum of expected discounted dividends, and define the fundamental house prices as the sum of discounted future cash flows. For example, Hott and Monnin (2008) define the fundamental house prices as the sum of the discounted future imputed rents. Black *et al.* (2006) assume the fundamental house prices is a constant proportion of the expected value of future real disposable income.

Some papers determine the intrinsic values of house prices by carrying out a regression on some basic variables. For example, according to the basic Capozza-Helsley urban model, construction cost inflation, real income growth and changes in real after-tax interest rates could explain major variation in real house price change. This model is utilized to determine the equilibrium house value by Abraham and Hendershott (1996), Capozza *et al.* (2002) and Bourassa *et al.* (2001). Case and Shiller (2003) find that prices move very much in line with income in the majority of states. McCarthy and Peach (2002) use co-integration to construct the long-run equilibrium model of real house prices as two simultaneous equations: on the demand side, house price is a function of housing stock, income and user cost; on the supply side, house price is expressed as a function of investment rate and construction cost.

Due to the high transaction costs, heterogeneity and illiquidity of housing, the price adjustment toward intrinsic value would be a prolonged process. Only a few existing papers analyze the dynamic process of the deviation from the fundamental values. Abraham and Hendershott (1996) and Bourassa *et al.* (2001) define the deviations from the fundamental values as the mean reversion term (or bubble burster) and integrate this term in the regression model to describe the dynamics. Capozza *et al.* (2002) assert that the value changes of house price (or house price returns) are governed by three parts: serial correlation, reversion to the fundamental value, and immediate partial adjustment to fundamentals. Black *et al.* (2006) regard that the deviation term from the fundamental values as a linear function of income.

As for the house price index, we will investigate both the FHFA[21]'s House Price Index and S&P/ Case-Shiller's Home Price Index. These two indices are most widely accepted nowadays. Both are repeat sales indexes. S&P/ Case-Shiller indices are value-weighted, based on 10 or 20 metropolitan areas[22], available from 1987. The FHFA's indices are unit-weighted, based on the fifty states and Washington D.C., available from 1975. Moreover, the FHFA's House Price Index only uses the data based on Fannie Mae and Freddie Mac mortgages. The Case-Shiller's House Price Index obtains data from county assessor and recorder offices and therefore covers more houses in the specific areas.

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[21] The FHFA refers to the Federal Housing Finance Agency, which is previously called OFHEO (the Office of Federal Housing Enterprise Oversight).

[22] 10 metropolitan areas include Boston, Chicago, Denver, Las Vegas, Los Angeles, Miami, New York, San Diego, San Francisco, and Washington DC. 20 metropolitan areas also include Atlanta, Charlotte, Cleveland, Dallas, Detroit, Minneapolis, Phoenix, Portland (Oregon), Seattle, and Tampa

The basic method we employ to detect the intrinsic values of house prices is co-integration. Since most macro variables are non-stationary and integrated of order one, by investigating the linear combinations and making the residual series stationary, we could find the long term relations. The Engle-Granger two-step method is based on least square regression. We utilize this method, instead of asset pricing principles, due to the following reason: the basic principle of asset pricing should be suitable for each specific house or household. However, it is difficult to find the corresponding rent or income for each house. Since in this paper, we use the aggregate house price index as the measure of house prices, it is also hard to obtain the exactly matching rent index or income.

What we will do in this paper is to define five measures of housing intrinsic values considering different economic aspects as the possible criteria in order to determine whether the house prices are overvalued or undervalued. It is due to the complicated relations among the economic variables and, therefore, the difficulty to determine the intrinsic values of house prices based on only one measure. The five measures include measures based on demand side, based on supply side, based on both demand and supply side, based on a rent index, and the combined measure.

After determining the intrinsic values of house prices, we continue to investigate the short term dynamics based on dynamic adjustments following Capozza et al (2002) and error correction models. The serial correlation term (momentum effect) and the deviation term from the intrinsic value are examined most. Based on national data, the coefficients on the serial correlation term and on the deviation term are around 0.6-0.9 and 0.03 respectively. In the dynamic adjustment part, the coefficient on the immediate adjustment term is 0.1 on average. In the error correlation model part, we examine the short term effects of more variables, including mortgage-related variables.

Inspired by Abraham and Hendershott (1996) and Capozza *et al.* (2002), we look into the possible overshooting problem of house price returns. We analytically derive the consecutive house price returns after an unexpected shock at time 0 assuming the intrinsic house price return equals to zero. Then, by simulation, we demonstrate the possible effects of the coefficients of the serial correlation term, deviation term and immediate adjustment term.

We have two major contributions in this paper. First, we advocate multiple measures of intrinsic values of house prices, so that we could comprehensively investigate whether the house prices are overvalued or undervalued. Following this line, we propose a combined measure of intrinsic values, which is a linear combination of the four measures obtained on the basis of co-integration. Such a kind of measure could extenuate certain extreme situations based on a sole measure. Second, we analytically derive the conditions of overshooting assuming the zero intrinsic house price returns. Then, based on simulation, we explicitly explain the effects of all the coefficients: the coefficient on the serial correlation term delays "overshooting", but strengthens its magnitude; the coefficient on the deviation term speeds up "overshooting" and also enlarge its magnitude; the coefficient on the immediate adjustment postpones the occurrence of "overshooting" and mitigates its magnitude.

The structure of this chapter is as follows. Section II.2 describes different measures of intrinsic values of house prices. Section II.3 presents the short term dynamics of house price returns. Section II.4 discusses the conditions for house price returns to “overshoot”. Section II.5 summarizes our conclusions.

II.2. Intrinsic Values of House Price

In this section, we will utilize the methodology of co-integration to catch the intrinsic values of house prices and, based on the relations with different variables, define them as one of the following five measures.

Since nominal variables are the ones we observe directly, in the following analysis, we use nominal variables. In order to reflect the real values, we include the Consumer Price Index as the indicator of inflation.

II.2.1. Co-integration

Intrinsic values could refer to the long-term values of house prices. Since most macro variables are non-stationary, co-integration becomes a good way to check the long term relations. It is because co-integration could capture the long-run relation tying different variables together, although there may exist some short term deviations.

Engle and Granger (1987) define the concept of *degree of integration* of a variable. If one variable becomes stationary after being differenced d times, then it is said to be integrated of order d, or $I(d)$. Most macro variables are integrated of order 1.

For a vector of time series $Y = (y_1, y_2, \cdots, y_n)'$, where y_i *(i=1,2,…,n)* is integrated of order *1*, if there exists a vector α so that $z = \alpha' Y$ is stationary, then this $n \times 1$ vector Y is said to be co-integrated. The α is called the co-integrating vector, which reflects the long term relationship among the n variables.

According to Engle and Granger (1987), one easiest way to estimate α is an ordinary least squares regression, which is called the "co-integrating regression". Hamilton (1994) also states that the OLS estimate of the co-integrating vector is consistent, as long as the residual of the regression is stationary.

II.2.2. Measure 1

In the long run, home construction firms are free to enter and exit the housing market. So they would make zero profits. Then, on the supply side, the house price should co-move with the construction cost. That is

$$\ln H_t^* = \alpha_0 + \alpha_1 * \ln CC_t + \alpha_2 * \ln CPI_t + \varepsilon_t , \qquad \text{(II.1)}$$

where H_t^* represents the intrinsic values of the house price index, CC_t means the construction cost, and CPI_t denotes the consumer price index representing the inflation. ε_t refers to the residual part.

The coefficient on construction cost is expected to be positive.

II.2.3. Measure 2

Black *et al.* (2006) state that the intrinsic values of house prices are related with the expected value of future real disposable incomes:

$$H_t = \gamma E_t \sum_{i=1}^{\infty} \left(\frac{1}{\prod_{j=1}^{i} \left(1 + \rho_{t+j}^*\right)} \right) Inc_{t+i} ,$$

where ρ_{t+j}^* is the discount rate.

On the demand side, household income and mortgage rate determine the housing affordability: increased household income and decreased mortgage rate enhanced the affordability. Therefore we could express the relationship as:

$$\ln H_t^* = \beta_1 * \ln Inc_t + \beta_2 * MR_t + \beta_3 * \ln CPI_t + \varepsilon_t, \qquad \text{(II.2)}$$

where Inc_t refers to the disposable personal income and MR_t refers to the 30-year fixed mortgage rate. So the coefficient on income should be positive, while that on the mortgage rate should be negative.

II.2.4. Measure 3

In the long run, the housing intrinsic values are determined simultaneously both from the supply and the demand side. This measure could be established as a reduced form.

$$\ln H_t^* = \gamma_0 + \gamma_1 * \ln Inc_t + \gamma_2 * MR_t + \gamma_3 * \ln CC_t + \gamma_4 * \ln CPI_t + \varepsilon_t. \qquad \text{(II.3)}$$

The signs of the coefficients on construction cost, income and mortgage rate should be the same as the ones in Measure 1 or Measure 2.

II.2.5. Measure 4

Plenty of literature regard rent as the dividend of house and assume the housing intrinsic value equals to the present value of future rents:

$$H_t^* = \mathrm{Re}\,nt_t + \frac{E_t H_{t+1}^*}{(1 + d_t)},$$

where $\mathrm{Re}\,nt_t$ denotes the corresponding rent (index) at time *t*; $E_t H_{t+1}^*$ refers to the expected value of future housing intrinsic values and d_t is the discount rate.

Following this idea, the housing intrinsic value should have the long run relationship with rent.

$$\ln H_t^* = \delta_0 + \delta_1 * \ln \mathrm{Re}\, nt_t + \delta_2 * \ln CPI_t + \varepsilon_t \,. \qquad \text{(II.4)}$$

And since house prices are increased with rents, the coefficient on rent is expected to be positive.

II.2.6. Measure 5

A measure combining all of the above effects may be needed. One way is to perform a regression of house price on all of the above variables (income, mortgage rate, construction cost, rent and consumer inflation index). The coefficient on income under such a kind of regression is significantly negative, which is opposite to what we expected. It may be due to multicollinearity. This indicates that it is not a good way to do the regression directly.

Another method is that we may assign a weight for each measure and obtain a weighted average. One possible weight is based on the inverse of volatility (standard deviation) of the differences between the actual and fitted log house price returns

$$\frac{1}{Std\left(\ln H - \ln H_i^*\right)},\ \left(i = 1,2,3,4\right).$$

So the corresponding weight is

$$\frac{\dfrac{1}{Std\left(\ln H - \ln H_i^*\right)}}{\displaystyle\sum_{i=1}^{4} \frac{1}{Std\left(\ln H - \ln H_i^*\right)}} \,. \qquad \text{(II.5)}$$

II.2.7. Empirical Results

We use the Construction Price Indices from the U.S. Census Bureau as construction cost. The disposable personal income is obtained from Bureau of Economic Analysis (BEA). The 30-year fixed mortgage rates come from the Federal Reserve Board. The rent index and Consumer Price Index are from U.S. Bureau of Labor Statistics (BLS). All of the above variables plus two house price indexes are integrated of order one, based on the Augmented Dickey-Fuller Unit Root Tests.

The residuals of the first four measures are stationary, according to the Augmented Dickey-Fuller Unit Root Tests (Table II.1). Therefore equation (II.1)--(II.4) reflect the co-integrating relations between the log house price indexes and the corresponding variables. These results may give some explanations about why plenty of papers use the rent-to-price and income-to-price ratio to measure whether there exists a bubble in the housing market.

For the first four measures and both house price indices, the coefficients on almost all of the variables have expected signs and reasonable magnitudes. Most of the coefficients are statistically and economically significant, except mortgage rate in Measure 3.

It is noticed that the log Consumer Price Index (CPI) has negative coefficients in every regression. This variable could be regarded as an adjustment from nominal variables to real variables. Since most of the coefficients on other independent variables are greater than one, it is reasonable to have negative coefficient on the log CPI.

Table II.1: Housing Intrinsic Value Measures

Based on the OLS estimation, the residuals are stationary. So under each of the four measures, either house price index is cointegrated with the corresponding variables. And each measure gives out a method of obtaining housing intrinsic values.

Dependent Variable: log house price index	the FHFA's HPI	the Case-Shiller's HPI
Independent Variable	Measure (1)	
log construction cost	1.331*** (0.030)	3.589*** (0.181)
log consumer price index	-0.054** (0.026)	-2.846*** (0.251)
Adj R-Sq	0.99996	0.958
ADF Tau	-3.51***	-2.06**

Independent Variable	Measure (2)	
log income	2.179*** (0.211)	4.886*** (0.430)
30-year fixed mortgage rate	-0.007** (0.003)	-0.028*** (0.010)
log consumer price index	-2.681*** -0.368	-7.469*** (0.752)
Adj R-Sq	0.9998	0.999
ADF Tau	-3.09***	-1.95**

Independent Variable	Measure (3)	
log income	0.313*** (0.116)	0.617 (0.430)
30-year fixed mortgage rate	-0.006** (0.003)	0.003 (0.014)
log construction cost	1.466*** (0.078)	3.275*** (0.284)
log consumer price index	-0.894*** (0.161)	-3.559*** (0.599)
Adj R-Sq	0.994	0.958
ADF Tau	-2.24**	-2.12**

Independent Variable	Measure (4)	
log rent index	1.735*** (0.257)	8.226*** (0.385)
log consumer price index	-0.480 (0.300)	-7.452*** -0.392
Adj R-Sq	0.962	0.999
ADF Tau	-2.73***	-2.59***

Note: *** represent 1% significance level; ** represent 5% significance level;
* represents 10% significance level

Figure II.2 demonstrates that, by the second quarter of 2008, the FHFA's House Price Indexes were still overvalued according to all of the five measures, but had different magnitudes. In the second quarter of 2008, based on the five measures, the FHFA's House Price Indexes were overvalued in the range of 5-10 percent.

The starting points of the latest overvaluation period were varied among the measures. Among the five measures, the starting points of the latest overvaluation period varied from 2003 till 2005. This result could explain why a lot of people investigated whether there were house bubbles since 2004.

During the latest overvaluation period, Measure 2 and 4 display that, in 2006, the FHFA's index was in excess of the intrinsic value by even above 15 percent. However, after 2007, the diversities between the five measures have decreased.

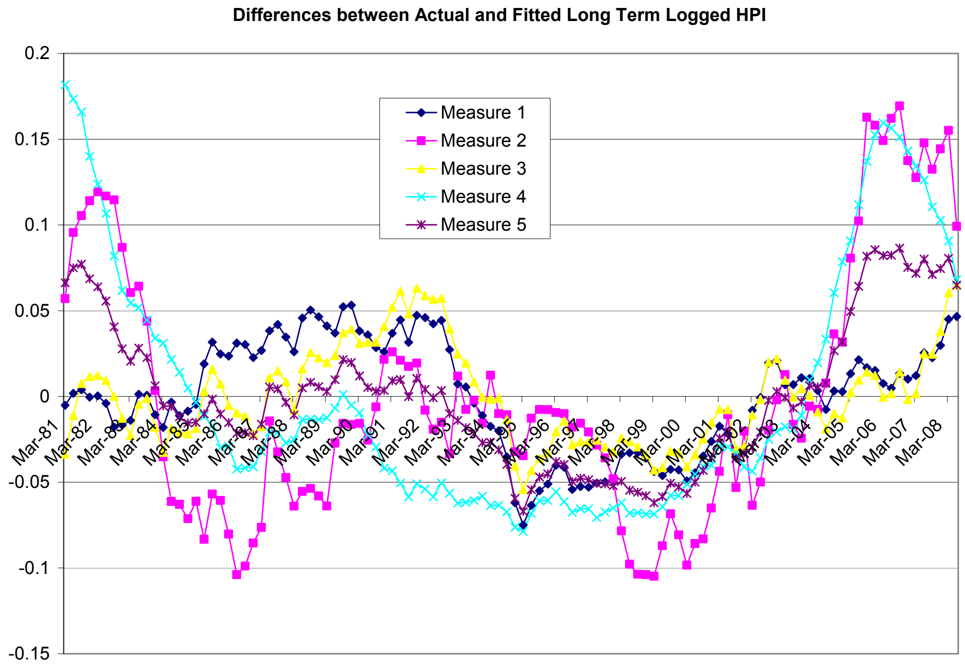

Figure II.2: Residuals between the Actual Log the FHFA's HPI and the Fitted Housing Intrinsic Values via Five Measures. $Residual = Actual - Fitted$. In the second quarter of 2008, based on the five measures, the FHFA's House Price Indexes were overvalued in the range of 5-10 percent. The starting points of the latest overvaluation period were varied among the measures. However, after 2007, the diversities between the five measures have decreased.

Figure II.3 shows that, at the second quarter of 2008, the Case-Shiller's House Price Index is overvalued only according to Measure 1 and 3, and all of the rest measures display an undervalued index. The intrinsic values determined by Measure 1 and 3 demonstrate that the index began to be undervalued from the fourth quarter of 2006 and then began to overvalued since 2008. Measure 2, 4 and 5 exhibit the undervaluation since 2008.

As for the recent overvaluation period, According to Measure 1 and 3, the Case-Shiller's House Price Index began to be overvalued from the second quarter of 2002. Measure 2, 4 and 5 reflect that the Case-Shiller's House Price Index began to be overvalued from the second quarter of 2004.

During the latest overvaluation period, the Case-Shiller's index has been in excess of the intrinsic value determined by Measure 2 and 4 by even above 20 percent from the third quarter of 2005 till the third quarter of 2006. The maximum excess of the Case-Shiller's index over the intrinsic values determined by Measure 5 is about 20 percent in the fourth quarter of 2005.

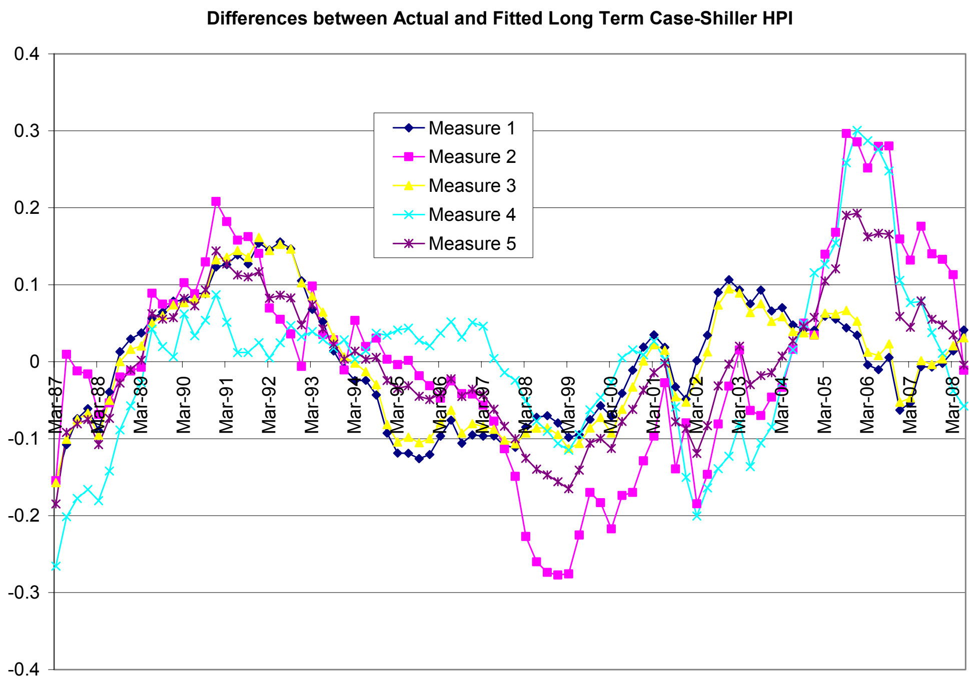

Figure II.3: Residual between the Actual Log Case-Shiller's HPI and the Fitted Housing Intrinsic Value via Five Measures. $Residual = Actual - Fitted$. The intrinsic values determined by Measure 1 and 3 demonstrate that the index began to be undervalued from the fourth quarter of 2006 and then began to overvalued since 2008. Measure 2, 4 and 5 exhibit the undervaluation since 2008.

As we have mentioned before, the diversities between the above results mainly come from the statistical differences between the FHFA's and Case-Shiller's HPI. Just like that it is hard to say which index is better, we can not tell which measure is best to indicate the intrinsic house values. Actually, it is more suitable to study all of the indicators in order to get a full view of the economy.

II.3. Short Term Adjustment and Housing Dynamics

The level relationship only reflects a long-term relationship and does not hold at all times. House prices would always deviate from their intrinsic values, although there is a tendency to move towards the intrinsic values. In this part, we investigate the short run relationships and dynamics.

II.3.1. Dynamic Adjustment

Following Capozza *et al.* (2002), the short term house price changes (or house price return), $HR_t = \ln H_t - \ln H_{t-1}$, may be affected by three parts:

$$HR_t = \alpha_0 + \alpha HR_{t-1} + \beta\left(\ln H_{t-1}^* - \ln H_{t-1}\right) + \gamma\left(\ln H_t^* - \ln H_{t-1}^*\right). \tag{II.6}$$

The first item, HR_{t-1}, refers to the serial correlation (momentum part). It is also called "bubble builder", which has the tendency to build a bubble.

The second item, $\left(\ln H_{t-1}^* - \ln H_{t-1}\right)$, is the deviation term from the intrinsic value and exhibits the partial adjustment to the deviation of house prices from the intrinsic values. This term has the tendency to burst the bubble.

The third term, $\left(\ln H_t^* - \ln H_{t-1}^*\right)$, is the immediate adjustment of the intrinsic values and presents the partial adjustment to contemporaneous intrinsic value changes.

Table II.2 and II.3 record the coefficients on short term dynamics of house price returns for the two indices. The coefficients on the lagged house price returns are positive, and economically as well as statistically significant. For the FHFA's index, the coefficients are around 0.6, while, for the Case-Shiller's index, they are around 0.9.

The coefficients on the deviation of house prices from the intrinsic values are around 0.03. Abraham and Hendershott (1996) obtain an estimation of 0.05 for all their 30 metropolitan statistical areas (MSAs). When the areas are divided into two parts, the estimate is 0.10 for the 14 coastal areas versus -0.005 for the 16 inland areas. Our estimations are roughly consistent with the previous research.

The coefficients on the immediate adjustment of the intrinsic values fluctuate a lot among different measures, ranging from 0.02 to 0.20. The relatively lower coefficients demonstrate that the house market is not an efficient market.

The empirical results suggest strong serial correlation and slow responses to the intrinsic values of house prices. The results are consistent with the previous real estate literature. So the house prices would take a long time to converge to their intrinsic values.

Table II.2: Short Term Dynamics of the FHFA's house price returns

This short term dynamics of the FHFA's house price returns are based on equation (II.6) and investigate the coefficients on the three parts.

Dependent Variable: house price return (the FHFA's index)					
Variable	Measure 1	Measure 2	Measure 3	Measure 4	Measure 5
Intercept	0.0023**	0.0034***	0.0032***	0.0008	0.0019
	(0.0011)	(0.0011)	(0.0010)	(0.0018)	(0.0012)
lagged house price return	0.5742***	0.6581***	0.5229***	0.6969***	0.6589***
	(0.0790)	(0.0771)	(0.0730)	(0.0781)	(0.0752)
deviation from intrinsic value	0.0352**	0.0198**	0.0689***	0.0168*	0.0241**
	(0.0173)	(0.0080)	(0.0199)	(0.0091)	(0.0110)
change of intrinsic value	0.2233***	0.0408	0.2080***	0.1986*	0.1662***
	(0.0613)	(0.0289)	(0.0440)	(0.1053)	(0.0623)

Note: *** represent 1% significance level; ** represent 5% significance level; * represents 10% significance level

Table II.3: Short Term Dynamics of the Case-Shiller's house price return

This short term dynamics of the Case-Shiller's house price returns are based on equation (II.6) and investigate the coefficients on the three parts.

Dependent Variable: house price return (the Case-Shiller's index)					
Variable	Measure 1	Measure 2	Measure 3	Measure 4	Measure 5
Intercept	-0.0008	-0.0004	-0.0008	-0.0008	-0.0007
	(0.0009)	(0.0009)	(0.0009)	(0.0009)	(0.0009)
lagged house price return	0.9241***	0.9340***	0.9175***	0.9587***	0.9308***
	(0.0491)	(0.0406)	(0.0481)	(0.0395)	(0.0414)
deviation from intrinsic value	0.0115	0.0228***	0.0138	0.0254***	0.0305***
	(0.0101)	(0.0060)	(0.0101)	(0.0076)	(0.0092)
change of intrinsic value	0.0778**	0.0189	0.0846***	0.0333	0.0551
	(0.0319)	(0.0165)	(0.0317)	(0.0237)	(0.0284)

Note: *** represent 1% significance level; ** represent 5% significance level; * represents 10% significance level

II.3.2. Error Correction Models

The Granger Representation Theorem states that, provided that time series are cointegrated, the short-term disequilibrium relationship can always be expressed in the error correction form.

Following the error correction model (ECM), we could check the effects of other variables on the house price returns.

We consider both the basic economic variables, such as the change of income and the change of construction cost, and the mortgage-related variables, such as the change of default rate.

$$HR_t = \alpha HR_{t-1} + \beta\left(\ln H_{t-1}^{*} - \ln H_{t-1}\right) + \gamma_0 + \gamma_1 \Delta Inc_t + \gamma_2 \Delta CC + \gamma_3 Inf_t$$
$$+ \gamma_4 \Delta \mathrm{Re}\, nt_t + \gamma_5 \Delta MR_t + \gamma_6 \Delta D_t + \gamma_7 Mar_t + \varepsilon_t \qquad \text{(II.7)}$$

ΔD_t refers to the change of default rate and the data source is the Mortgage Bankers Association (MBA). Mar_t stands for the margin, which is the residual of 30-year fixed-rate mortgage rate deducted by 10-year treasury bond rate.

Table II.4 exhibits the regression results on the FHFA's house price returns. And Table II.5 displays the results on the Case-Shiller's house price returns.

The coefficients on lagged house price returns and deviations from intrinsic values for both the indexes are similar to the coefficients based on equation (II.6).

As for the other variables, from Table II.4, all the coefficients have the expected signs. The change of log rent index, change of log construction cost, and change of log income are positively correlated with the FHFA's house price return. The change of mortgage rate, change of default rate, and margin are negatively correlated with the house price return. The inflation rates are an adjustment from the nominal variables to the real variables and so their sign of coefficients is related with the other variables in the equation.

The regression results on the Case-Shiller's house price returns have some exceptions. The coefficients on the change of income are negative, although they are statistically insignificant. The coefficients on the change of mortgage rate are positive, although insignificant. There may exist several reasons. First, the Case-Shiller's house price index is from 1987. Compared between the two periods 1975-2007 and 1987-2007, mortgage rate graphs have changed a lot. Especially from 1987 till 2007, the mortgage rates had the roughly downward trend. Second, the Case-Shiller's index has some statistical differences from the FHFA's index, as we mentioned before. Third, the short term relations may have some random effects which have impacts on the regression results. We chart the two index returns and change of mortgage rate in Figure II.4. It is obvious that the FHFA's house price returns have negative relations with the change of mortgage rate, while the Case-Shiller's house price returns do not, especially for the recent few years.

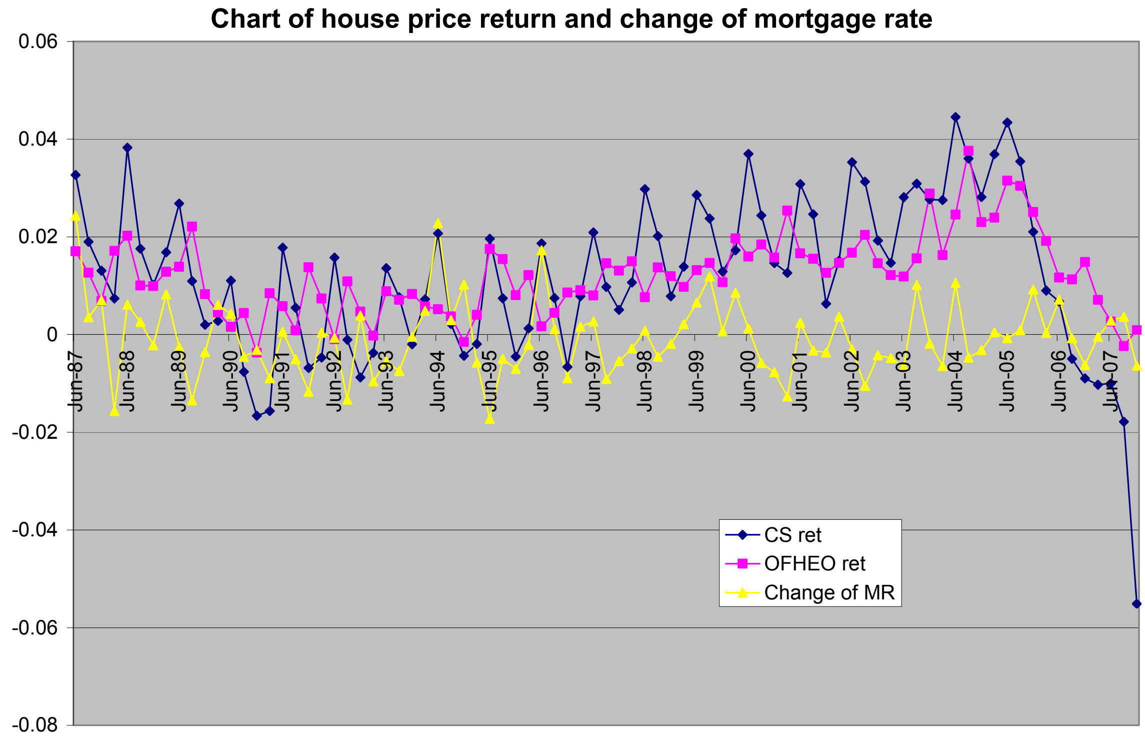

Figure II.4: Chart of the Case-Shiller's and the FHFA's house price returns and changes of mortgage rates

This chart is used to compare the relationship between the FHFA's house price returns and the changes of mortgage rate and the relationship between the Case-Shiller's house price returns and the changes of mortgage rate. the FHFA's house price returns have negative relations with the change of mortgage rate, while the Case-Shiller's house price returns do not, especially for the recent few years. It may explain why the coefficient on the change of mortgage rate in equation (II.7) for the Case-Shiller's returns is positive, which is opposite to the coefficient for the FHFA's returns.

Table II.4: House Price (the FHFA's Index) Dynamics with more variables
This table exhibits the short term dynamic results based on equation (7). Dependent Variable is the FHFA's House Price Return. Basically all the coefficients have the expected sign.

Variable	Measure 1	Measure 2	Measure 3	Measure 4	Measure 5
Intercept	0.0034 (0.0025)	0.0034 (0.0025)	0.0044* (0.0023)	0.0043 (0.0029)	0.0032 (0.0026)
lagged house price return	0.6295*** (0.0796)	0.6159*** (0.0802)	0.5706*** (0.0775)	0.6225*** (0.0815)	0.6253*** (0.0797)
deviation from intrinsic value	0.0289* (0.0172)	0.0114 (0.0084)	0.0723*** (0.0204)	0.0005 (0.0102)	0.0141 (0.0125)
change of log rent index	---	---	---	0.2973 (0.2234)	0.3343 (0.2112)
change of log construction cost	0.2476*** (0.0833)	0.2393*** (0.0836)	0.2536*** (0.0795)	0.2497*** (0.0841)	0.2500*** (0.0835)
change of log income	0.0202 (0.0650)	0.0289 (0.0657)	0.0179 (0.0621)	0.0059 (0.0678)	0.0006 (0.0675)
inflation rate	-0.1292 (0.1272)	-0.1661 (0.1238)	-0.0953 (0.0012)	-0.2081 (0.1279)	-0.1761 (0.1264)
change of mortgage rate	-0.0036*** (0.0013)	-0.0033*** (0.0013)	-0.0039*** (0.0012)	-0.0034** (0.0013)	-0.0033** (0.0013)
change of default rate	-0.0100 (0.0109)	-0.0063 (0.0116)	-0.0062 (0.0105)	-0.0110 (0.0117)	-0.0063 (0.0116)
margin	-0.0005 (0.0013)	-0.0003 (0.0013)	-0.0009 (0.0012)	-0.0020 (0.0016)	-0.0018 (0.0016)

***1% significance level; ** 5% significance level; * 10% significance level

Table II.5: House Price (the Case-Shiller's Index) Dynamics with more variables
This table exhibits the short term dynamic results based on equation (II.7). Dependent Variable is the Case-Shiller's House Price Return. Basically all the coefficients have the expected sign, except the change of mortgage rate.

Variable	Measure 1	Measure 2	Measure 3	Measure 4	Measure 5
Intercept	0.0082	0.0122**	0.0081	0.0128**	0.0104*
	(0.0057)	(0.0052)	(0.0057)	(0.0051)	(0.0052)
lagged house price return	0.9568***	0.8967***	0.9491***	0.9076***	0.8989***
	(0.0556)	(0.0516)	(0.0561)	(0.0501)	(0.0528)
deviation from intrinsic value	0.0028	0.0248***	0.0079	0.0323***	0.0346***
	(0.0104)	(0.0060)	(0.0106)	(0.0074)	(0.0094)
change of mortgage rate	0.0031	0.0029	0.0029	0.0032	0.0026
	(0.0023)	(0.0020)	(0.0023)	(0.0020)	(0.0021)
change of log construction cost	0.0968	0.1673	0.1168	0.2164*	0.2143*
	(0.1375)	(0.1207)	(0.1375)	(0.1214)	(0.1262)
inflation rate	-0.4964**	-0.3617	-0.4844**	-0.5184***	-0.4029**
	(0.1923)	(0.1751)	(0.1911)	(0.1697)	(0.1770)
change of default rate	-0.0122	-0.0005	-0.0118	-0.0004	-0.0027
	(0.0157)	(0.0145)	(0.0156)	(0.0143)	(0.0147)
margin	-0.0034	-0.0064	-0.0034	-0.0064*	-0.0054*
	(0.0034)	(0.0032)	(0.0034)	(0.0031)	(0.0032)

***1% significance level; ** 5% significance level; * 10% significance level

II.4. Overshooting

Abraham and Hendershott (1996) and Capozza *et al.* (2002) have argued that the inefficiency in the housing market may cause house price returns in some areas to significantly "overshoot" the fundamental values.

One famous overshooting model was built by Rudiger Dornbusch (1976). In Dornbusch's model, assuming rigidity of domestic prices, an unanticipated permanent increase in money supply will cause the initial depreciation of the exchange rate to be larger than long-run depreciation. That is, the exchange rate must overshoot first in order for the ensuing appreciation to clear bond and money markets.

The "overshooting" concept in housing market means that, if house price returns are overvalued compared with the fundamental values (determined by the intrinsic values of house prices), they may be adjusted to a level lower than the fundamental values first, and then back to the fundamental values gradually. In order to be "overshot", the mutual functions of serial correlation term and deviation term from the intrinsic values are needed. Abraham and Hendershott (1996) and Capozza *et al* (2002) have utilized simulation methods to obtain the overshooting path and the rough combination of the two terms. In this paper, we will try to derive the rough range of the coefficients to "overshoot" and clarify the role of each term.

II.4.1. Case 1

First assume the intrinsic value of house prices is constant over time, that is $\ln H_t^* = \ln H^* = c$. This is the simplest case. Then the house price return would be obtained from the following equation, transformed from equation (II.6):

$$HR_t = \alpha HR_{t-1} + \beta(c - \ln H_{t-1}) + \varepsilon_t$$

$$= \alpha HR_{t-1} + \beta\left(-\sum_{i=1}^{t-1} HR_i\right) + \varepsilon_t \text{[23]}. \qquad \text{(II.8)}$$

Suppose there is a shock $\varepsilon_0 = 1$ (unit) (e.g. 10 percent) at time 0, so that $HR_0 = 1$ (unit) and all the rest innovations are zero. We could check the impacts on HR_t through the serial correlation term and the deviation term. Table II.6 exhibits the impulse response functions[24] for HR_t expressed by α and β. Table II.7 displays the range of β for some α's in order to overshoot at the specific time period[25]. In Table II.7, we select 5 scenarios: $\alpha = 0.1$, $\alpha = 0.5$, $\alpha = 0.6$, $\alpha = 0.7$, and $\alpha = 0.9$.

Table II.6: the impulse response functions for HR_t

Suppose there is a shock $\varepsilon_0 = 1$ (unit) at time 0 and all the rest innovations are zero.

The impulse response functions for HR_t are expressed by α and β.

Period *t*	HR_t
1	$\alpha - \beta$
2	$(\alpha - \beta)^2 - \beta$
3	$(\alpha - \beta)^3 - \beta - 2\beta(\alpha - \beta)$
4	$(\alpha - \beta)^4 - 3\beta(\alpha - \beta)^2 - \beta + \beta^2 - 2\beta(\alpha - \beta)$
5	$(\alpha - \beta)^5 - 4\beta(\alpha - \beta)^3 - 3\beta(\alpha - \beta)^2 + 3\beta^2(\alpha - \beta) - 2\beta(\alpha - \beta) - \beta + 2\beta^2$
6	$(\alpha - \beta)^6 - 5\beta(\alpha - \beta)^4 - 4\beta(\alpha - \beta)^3 - 3\beta(\alpha - \beta)^2 - 2\beta(\alpha - \beta)$ $- 6\beta^2(\alpha - \beta)^2 - 2\beta^2(\alpha - \beta) - \beta + 3\beta^2 - \beta^3$

[23]We assume $\ln H_t = \ln H_0 + \sum_{i=1}^{t} HR_i = c + \sum_{i=1}^{t} HR_i$

[24] Impulse-response functions are built on vector autocorrelation regression (VAR) and describe the response of one variable to a one-time shock. So they could be utilized to measure the persistence and magnitude of a previous shock. Please check Hamilton (1994) as a reference.

[25] We may regard this period as one quarter in order to be consistent with our previous estimation.

From Table II.7, we could find that, basically with the increase of α, β should increase correspondingly in order to get "overshooting". For a specific α, the higher β is, the earlier "overshooting" occurs.

Table II.7: the possible combinations of α and β for the occurrence of overshooting at each period[26]

This table displays the range of β for some α's in order to overshoot at the specific time period. 5 scenarios are selected: $\alpha = 0.1$, $\alpha = 0.5$, $\alpha = 0.6$, $\alpha = 0.7$ and $\alpha = 0.9$.

	$\alpha = 0.1$	$\alpha = 0.5$	$\alpha = 0.6$	$\alpha = 0.7$	$\alpha = 0.9$
t=1	$0.1 < \beta$	$0.5 < \beta$	$0.6 < \beta$	$0.7 < \beta$	$0.9 < \beta$
t=2	$0.008 < \beta$	$0.134 < \beta$	$0.178 < \beta$	$0.225 < \beta$	$0.328 < \beta$
t=3	$0.001 < \beta < 0.844$	$0.048 < \beta$	$0.072 < \beta$	$0.099 < \beta$	$0.162 < \beta$
t=4	$0.0001 < \beta$	$0.020 < \beta$	$0.033 < \beta$	$0.049 < \beta$	$0.089 < \beta$
t=5	$0 < \beta < 0.608$ or $0.944 < \beta$	$0.009 < \beta < 0.500$	$0.017 < \beta < 0.525$	$0.028 < \beta < 0.559$	$0.061 < \beta < 0.642$
t=6	$0 < \beta < 0.257$ or $0.767 < \beta$	$0.004 < \beta < 0.452$	$0.009 < \beta < 0.519$	$0.016 < \beta < 0.592$	$0.039 < \beta < 0.753$

Figure II.5 and II.7 display the different paths of house price returns with variable α and β. With the same β and the higher α, "overshooting" occurs relatively later and its magnitude is enlarged, which shows the strong momentum effects. For example, if $\alpha = 0.1$ and $\beta = 0.03$, the house price return becomes negative at period *2* and the lowest return is -0.03 unit at period 3. If $\alpha = 0.9$ and $\beta = 0.03$, the house price returns are positive until period *7* and the lowest return is -0.5 unit at period 14. Hence the strong momentum term will impede the mean reversion process.

With the same α and the higher β, "overshooting" occurs relatively earlier and house price returns move back to the equilibrium value more quickly, which shows the strong

[26] Theoretically β is expected to be in the range [0,1]. Empirically β may also be less than 0. Abraham and Hendershott (1996) obtained $\beta = -0.005$ for the sample of 16 inland. Basically $\beta < 0$ means that this deviation term strengthens the momentum effect. So we assume β is always in [-1,1].

mean reversion (bubble buster) effects. Figure II.5 exhibits that, for $\alpha = 0.6$ and $\beta = 0.09$, house price returns "overshoot" since the third quarter[27] and are almost back to the equilibrium value after about 4 years. While, for $\alpha = 0.6$ and $\beta = 0.01$, house price returns begin to overshoot from the sixth quarters and need at least 10 years to be almost back to the equilibrium value. The higher β also augments "overshooting" magnitude. Figure II.5 demonstrates that the lowest returns for $\beta = 0.01$, $\beta = 0.03$, and $\beta = 0.09$ are -0.05 unit at period 14, -0.12 unit at period 9, and -0.25 unit at period 6, respectively.

Figure II.6 and II.8 exhibit the different paths of house prices with variable α and β. Based on our assumption, house prices should go back toward their intrinsic values, which is one here. Before reaching the intrinsic value, different parameters determine variety of paths. In the most possible case $\alpha = 0.6$ and $\beta = 0.03$, for a 10 percent shock of house price return at time 0, the house prices appreciate 22 percent after 1.25 years and then last about another 7 years to move back to the reasonable range around the intrinsic value. For $\alpha = 0.6$ and $\beta = 0.01$, the house prices obtain a 25 percent bubble after about 1.5 years and then the bubble bursts slowly. For $\alpha = 0.6$ and $\beta = 0.09$, the house prices only magnify by 18 percent and are back to the intrinsic value after about 3 years.

From Figure II.8, it can be found that, with the same β and the higher α, the house price would form the bigger bubble and then move back to the intrinsic value quicker. Under the extreme situation $\alpha = 0.9$, after first reaching the intrinsic value, house prices would even continue to go down.

Above all, both α and β magnify the magnitude of "overshooting" for house price returns. However, α tends to delay the occurrence of overshooting, while β tends to speed

[27] We assume each simulation tick is one quarter.

it up. Due to the mean reversion effects of higher β, even if the magnitude for house price returns to "overshoot" is increased, the house price bubble may not be expanded.

II.4.2. Case 2

Now we assume the intrinsic values of house prices increase with a constant number over time, that is $\ln H_t^* - \ln H_{t-1}^* = HR_t^* = c$. Then, based on equation (II.6), the house price returns under the dynamic adjustment would become:

$$HR_t = \alpha HR_{t-1} + \beta\left(\ln H_{t-1}^* - \ln H_{t-1}\right) + \gamma HR_t^* + \varepsilon_t$$

$$= \alpha HR_{t-1} + \beta\left(\sum_{i=1}^{t-1}\left(HR_i^* - HR_i\right)\right) + \gamma HR_t^* + \varepsilon_t \,. \qquad \text{(II.9)}$$

Suppose there is a shock $\varepsilon_0 = 10$ percent at time 0 and all the rest innovations are zero. Assume $HR_t^* = 1$ percent. It is needed to be clarified that "overshooting" means that the actual house price returns are less than HR^* and not necessarily less than zero. Figure II.9-II.14 demonstrate the impacts of variable parameters on house price returns and house prices.

Comparing Figure II.7 and II.9, we could see that the occurrence time of "overshooting" is similar. The one with less α still "overshoots" earlier. Figure II.11 displays similar results as Figure II.5

Figure II.10 and II.12 demonstrate that the appreciation magnitude of house prices becomes smaller, compared with the situations of Figure II.8 and II.6. For example, when $\alpha = 0.6$ and $\beta = 0.03$, for a 10 percent shock of house price return at time 0, the house price appreciates 8 percent, instead of 22 percent, after 1 years.

Figure II.13 and II.14 exhibit the effects of γ's. As γ becomes smaller, overshooting occurs earlier and the magnitudes are higher. We may regard γ as a criterion reflecting the efficiency of house market, so that greater γ stands for more efficiency. Then efficient market will have less overshooting phenomenon. Under the efficient market, house price will change quickly back to the intrinsic value.

II.5. Conclusion

This paper presents the possible measures of the intrinsic values of house prices, which stand for the long run equilibrium. Different measures may give different conclusions on whether the house prices are undervalued or overvalued in some specific period. At least from these estimations, we could learn when the over-price or under-price problem should be paid attention to.

As for the short term adjustment, the serial correlation term and deviation term from the intrinsic value play an important role on the dynamics of house price return.

Due to the inefficiency of housing market, house price returns may "overshoot". The timing and magnitude of "overshooting" highly depend on the coefficients of the three terms.

As for the further research, the intrinsic value and short term dynamics of local house prices should be investigated.

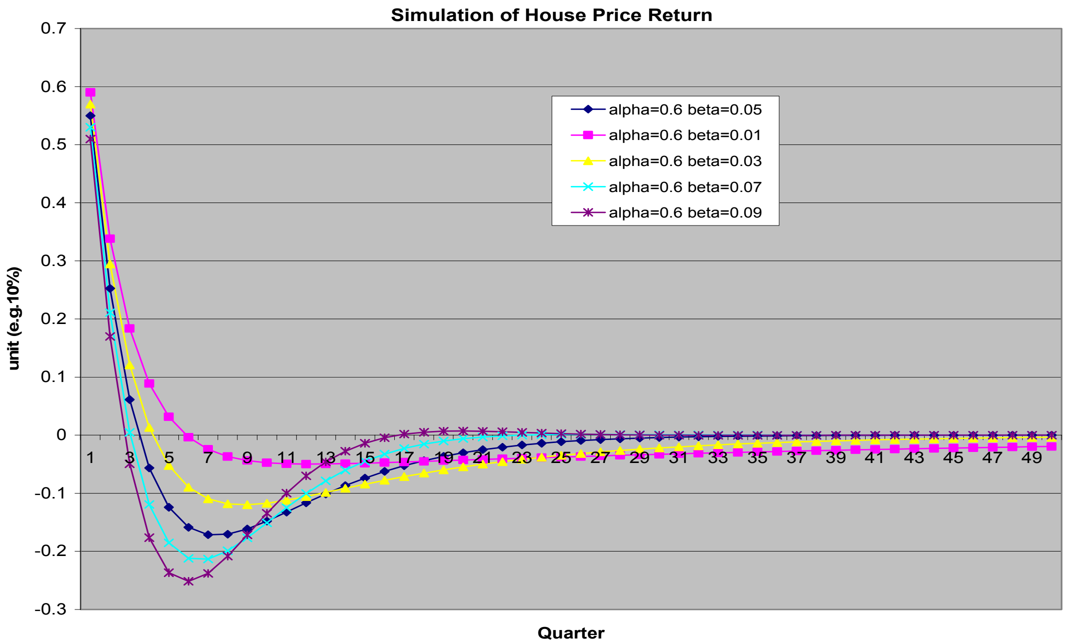

Figure II.5: Simulation of House Price Return Paths with the same α and different β's
With the same α and the higher β, "overshooting" occurs relatively earlier and house price returns move back to the equilibrium value more quickly, which shows the strong mean reversion (bubble buster) effects. The higher β also augments "overshooting" magnitude.

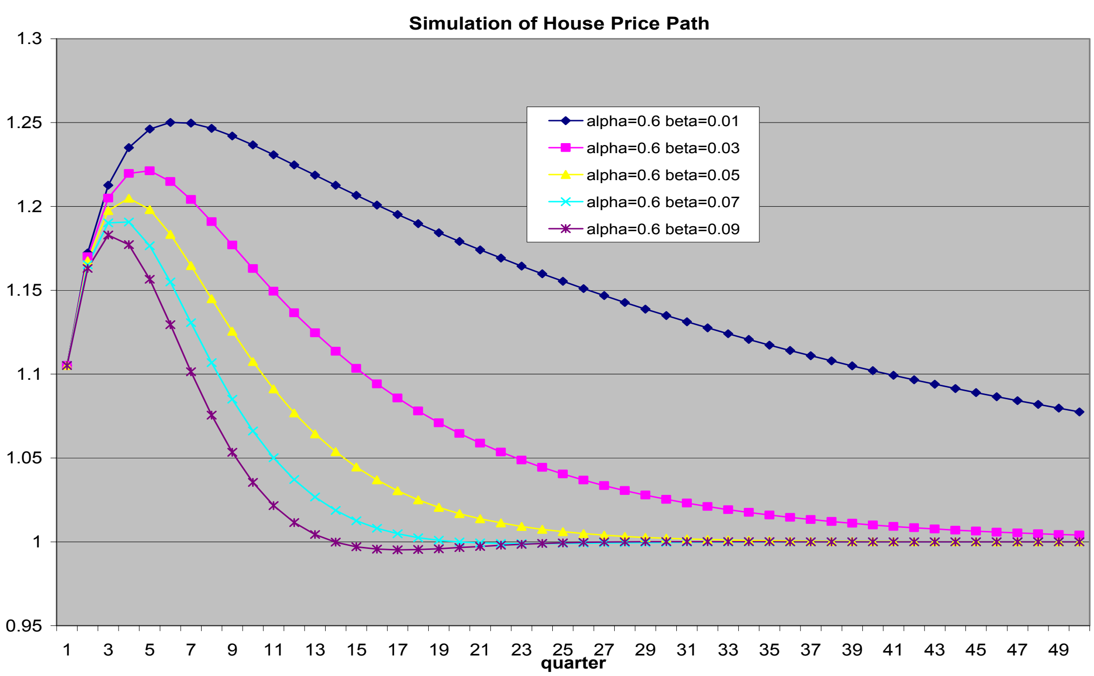

Figure II.6: Simulation of House Price Paths with the same α and different β's
With the same α and the higher β, the house price would form the smaller bubble and then move back to the intrinsic value quicker.

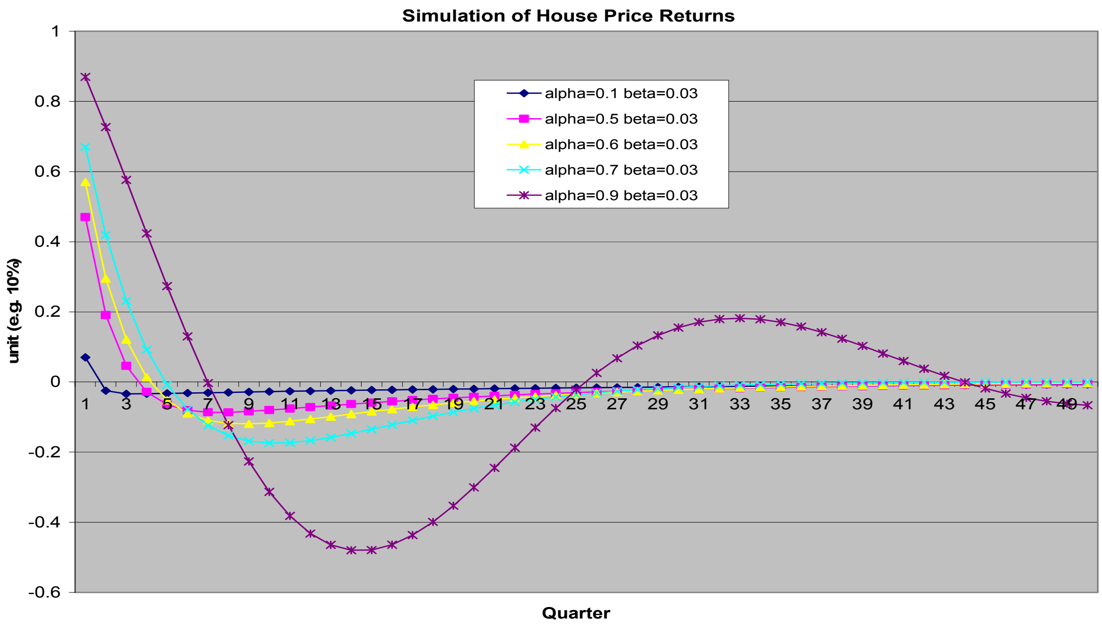

Figure II.7: Simulation of House Price Return Paths with the same β and different α's
With the same β and the higher α, "overshooting" occurs relatively later and its magnitude is enlarged, which shows the strong momentum effects.

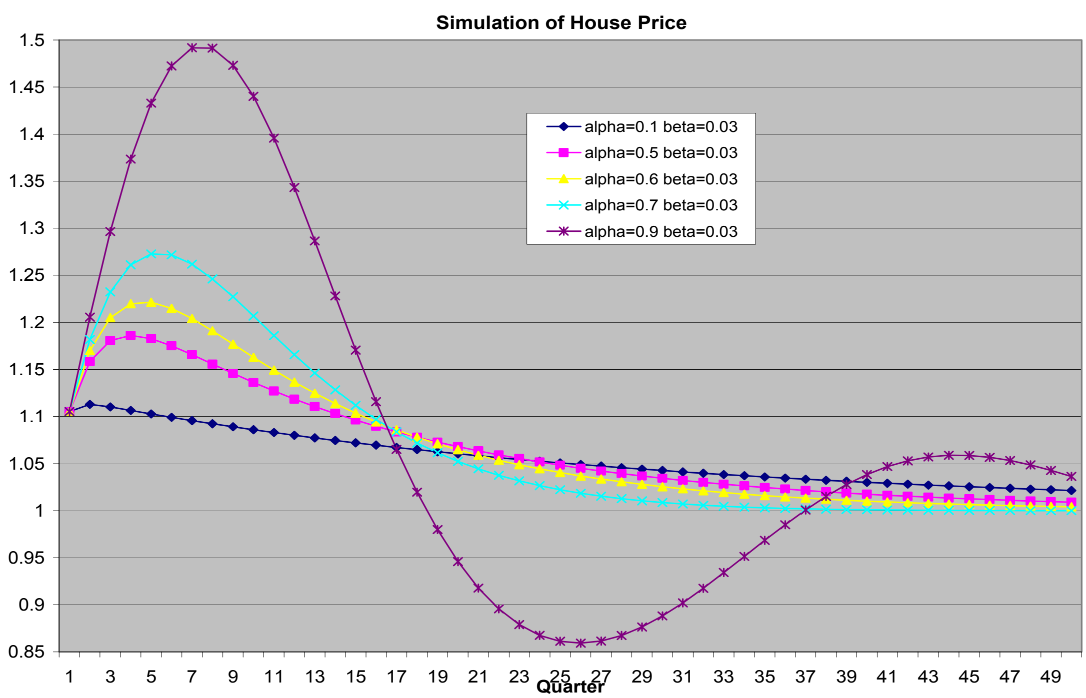

Figure II.8: Simulation of House Price Paths with the same β and different α's
With the same β and the higher α, the house price would form the bigger bubble and then move back to the intrinsic value quicker.

Figure II.9: Simulation of House Price Return Paths with the same β, γ and different α's
The results are similar with Figure II.7. With the same β and the higher α, "overshooting" occurs relatively later and its magnitude is enlarged.

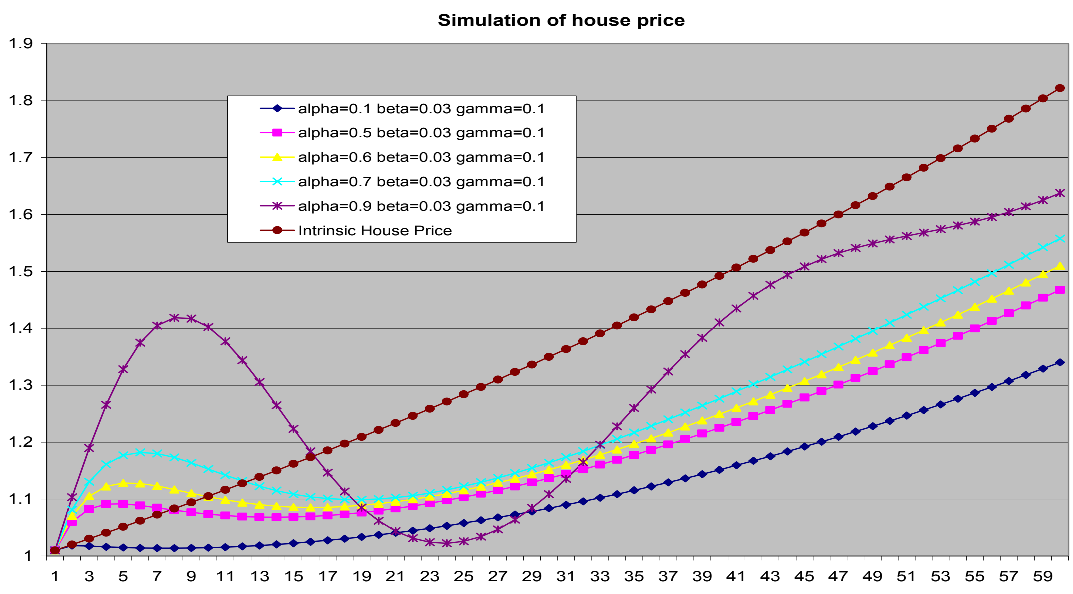

Figure II.10: Simulation of House Price Paths with the same β, γ and different α's
This figure demonstrates that the appreciation magnitude of house prices becomes smaller, compared with Figure II.8.

Figure II.11: Simulation of House Price Return Paths with the same α, γ and different β's
The results are similar with Figure II.7. With the same α and the higher β, "overshooting" occurs relatively earlier and house price returns move back to the equilibrium value more quickly. The higher β also augments "overshooting" magnitude.

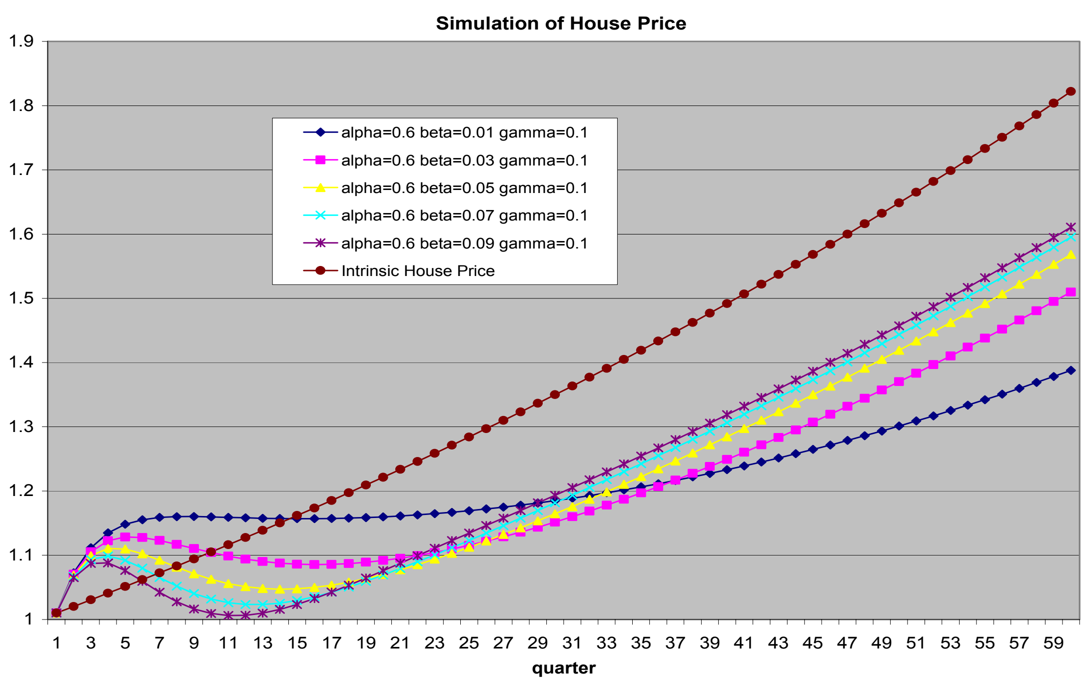

Figure II.12: Simulation of House Price Paths with the same α, γ and different β's
This figure demonstrates that the appreciation magnitude of house prices becomes smaller, compared with Figure II.6.

Figure II.13: Simulation of House Price Returns with the same α, β and different γ's
As γ becomes smaller, overshooting occurs earlier and the magnitudes are higher, which reflects a less efficient house market.

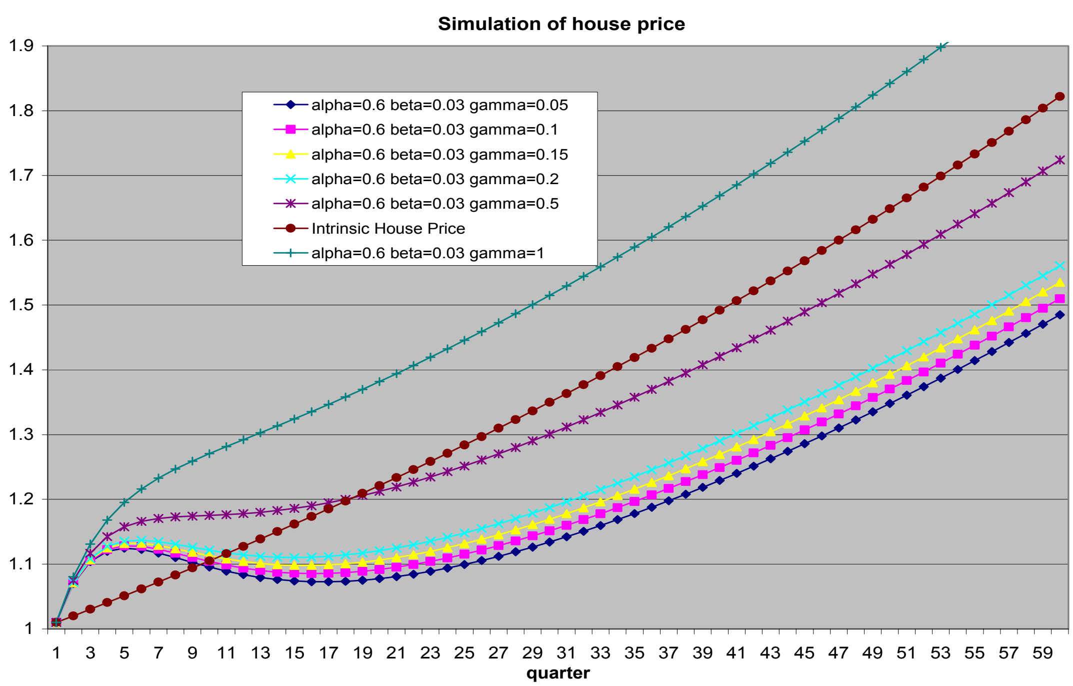

Figure II.14: Simulation of House Prices with the same α, β and different γ's
As γ becomes greater, house prices change quickly back to the intrinsic values, which reflects a higher efficient house market.

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Chapter III Bond Insurers: Avoiding Capital Pro-cyclicality

Abstract

This chapter describes the business features of bond insurers and the impacts of business cycles on them. A structural model with time-varying correlations, which are closely tied up with the business cycle, is adopted. When deriving the total loss distribution and economic capital for a bond insurer, we consider losses due to bond insurers' downgrading, and losses from both insurance contracts and investment portfolio. On that basis, we propose forward-looking smoothing rules of capital over a full business cycle, instead of only based on a short-term horizon, to avoid the procyclicality. The simulation results show the smoothed capital may vary from lower degree of procyclicality to totally counter-cyclicality, corresponding to the different parameter values. With the smoothed capital, a bond insurer can actually establish some capital buffer in good times to support the potential losses in crisis.

III.1. Introduction

Bond insurers[28], operated as monoline insurers since 1989, were some of the first financial institutions affected by the financial crisis beginning in 2007. As a result, credit rating agencies began downgrading most of the bond insurers. This triggered serious problems for bond insurers. Bond insurers essentially rented their AAA credit ratings to lower-rated debt. Without such ratings, not to mention the extreme difficulty for them to get new business, the value decrease of all the currently insured bonds would need higher loss reserve and capital from bond insurers. It is crucial to make sure bond insurers have enough capital to cover the potential losses.

One important feature of bond insurers is that their business is very sensitive to the business cycle. The bonds have the tendency to have higher correlations with each other in deep recession (crisis), so systemic economic shocks may have serious impacts on bond insurers. In order to keep their ratings, bond insurers need more capital in credit crisis, which may worsen their financial conditions further. Therefore, their capital requirements may be even more pro-cyclical.

Many aspects of credit insurance have been addressed in the literature, such as Merton (1977) and Gendron *et al.* (2006b). Especially, Lai and Soumare (2010) analyze the effects of time-varying correlations and loan maturities on the risk-based capital that backed credit insurance portfolios. Drake and Neale (2010) examine four primary risk

[28] Bond insurers are also called "financial guaranty insurance companies" or "financial guarantors".

exposures and the failings of the regulatory framework of these insurers. The pro-cyclicality of bond insurers' capital requirements has not been sufficiently investigated.

Plenty of papers examined the pro-cyclicality problem in other areas. Andritzky *et al.* (2009) discuss policies to mitigate pro-cyclicality in private sector risk management and regulation.

The pro-cyclicality problem in the new Basel Accord is widely discussed, such as Ervin and Wilde (2001), Cosandey and Wolf (2002), Purhonen (2002), Kashyap and Stein (2004), Gordy and Howells (2006), and Repullo *et al.* (2009). Kashyap and Stein (2004) point out that the target solvency probability in the regulatory rule should be time-varying over the cycle. Gordy and Howells (2006) explore several smoothing rules for capital. Repullo *et al.* (2009) assess the pro-cyclical effects of bank capital regulation in a dynamic equilibrium model.

In the area of bank loan loss reserves, since 2007, the Spanish model of dynamic provisioning has attracted attention, since it is regarded a countercyclical tool, compared with the prevailing incurred loss model. The papers include Balla and McKenna (2009) and Saurina (2009).

Our purpose is to examine the interaction between bond insurers' operations and financial crisis, and propose smoothing rules for bond insurers' capital calculation in order to avoid the procyclicality. We use a structural model with some extensions. Our paper is characterized in the following aspects: (1) time-varying bond correlations are defined over cycle, which is in line with Lai and Soumare (2010). (2) The business cycles represented by bond yield spread are identified, using the approximate band-pass filters proposed by Baxer and King (1999), since bond insurers' operations heavily depend on credit market. So we can classify the economy as crisis and non-crisis. (3) We analyze the impacts of bond insurers' downgrades on bond values and incorporate them into the total losses of bond insurers. (4) The losses on both insurance contracts and investment portfolios are considered. (5) We propose forward-looking smoothing rules of capital over a full business cycle, instead of only based on a short-term horizon. The simulation results show the effects of changing parameter values in smoothing rules. The smoothed capital may vary from lower degree of procyclicality to totally counter-cyclicality, corresponding to the different parameter values. With the smoothed capital, a bond insurer can actually establish some capital buffer in good times to support the potential losses in crisis.

The rest of the chapter is organized as follows. Section III.2 provides an overview of bond insurance industry. Section III.3 presents the model. Section III.4 identifies business cycle, using bond yield spread index. Section III.5 presents our simulation results. Section III.6 discusses the related references to establish contingent capital buffer. Section III.7 is the conclusion.

III.2. Bond Insurance Industry: Overview

In a bond insurance business, a bond issuer pays premiums to a bond insurer and obtains insurance from a bond insurer. As a result, in the event that the bond issuer defaults, the bond insurer will provide interest and principal repayments as specified. Its important effect is that the rating of the bond is raised to the rating of the bond insurer. Therefore a bond insurer must have almost perfect credit rating accordingly.

Table III.1 lists the main events occurred in the bond insurance industry. Since the establishment of the first bond insurer in 1971, bond insurers first focused on insurance on public finance. Later in the 1990s, the bond insurers were involved with structured financial products, such as mortgage-based Collateralized Debt Obligations (CDO). The involvement grew quite large by the early 2000s. Drake and Neale (2010) shows that nowadays bond insurers mainly offer four types of credit enhancement:

1. insurance on public finance
2. insurance on structured finance
3. credit default swaps (CDS)
4. guaranteed investment contracts (GICs)

According to the annual reports of bond insurers, insurance on public finance includes General Fund Obligation, Municipal Utilities, Transportation, Health Care, Higher Education, Municipal Housing, and other financing bonds.

Insurance on structured finance includes Collateralized Debt Obligations, Mortgage-Backed Residential, Mortgage-Backed Commercial, Consumer Asset Backed, and Corporate Asset Backed securities.

Credit Default Swaps are generally provided for protection on structured finance.

Table III.1: Significant events in the bond insurance industry

Source: Drake and Neale (2010): Financial Guarantee Insurance and the Failures in Risk Management

Year	Company events	Bond insurance events
1971	American Municipal Bond Assurance Corp. (AMBAC), the first bond insurer, established by MGIC	
1973	Municipal Bond Insurance Association (MBIA) created	
1982	Baldwin-United buys MGIC (including AMBAC subsidiary)	
1983	Financial Guarantee Insurance Company (FGIC) formed	Default of $2.25 billion Washington Public Power System bonds, insured by AMBAC. Baldwin-United (parent to AMBAC) goes bankrupt.
1985	Financial Security Assurance (FSA) founded Bond Insurance Guarantee founded Citicorp acquires AMBAC	
1986	Capital Guarantee founded	Financial Guaranty Insurance Model Act of 1986 passed by NAIC.
1988	FGIC acquired by General Electric Capital	25% of municipal bonds insured.
1989	FSAs acquired by US West Bond Insurance Guarantee (BIG) acquired by MBIA	New York implements law that financial guarantors must be monoline insurers and meet minimum capital requirements.
1991	AMBAC spun off by Citicorp	
1994		Averted crisis in bond market with Orange County bonds. NYSID [29] issues letter stating that synthetic guaranteed investment contracts are prohibited.
1995	FSA acquires Capital Guarantee	NYSID enables the monocline bond insurers to write insurance on Guaranteed Investment Contracts.
1997	American Capital Access (ACA) Financial Guarantee Corporation formed	NYSID allows monoline bond insurers to participate in credit default swaps.
1998	Capital Markets Assurance Corp. acquired by MBIA	
1999	CMAC merged with Amerin, forming Radian	NYSID issues letter clarifying that insolvency, payment default or downgrade of a security's credit rating are payment triggers under a CDS.
2000	FSA acquired by Groupe Dexia. XL Capital Assurance Inc., started in 2000 by XL Capital Ltd.	40% of municipal bonds insured. Commodity Futures Modernization Act, H.R. 5660, passes leaving the credit default market generally unregulated.
2001	CDCI IXIS Financial Guarantee (CIFG) founded as a subsidiary of Caisse des Depots	
2002	CIFG founded	NYSID withdraws prohibition on synthetic guaranteed investment contracts.
2004		54% of municipal bonds insured. The New York Department of Insurance revises the insurance code to redefine "asset-backed securities" to allow policies on CDS securities and pools.
2006	PMI Guarantee founded	
2007	Berkshire Hathaway establishes Berkshire Hathaway Assurance	
2008	Assured Guarantee acquires FSA.	Majority of bond insurers lose triple-A rating.
2009	MBIA restructures and renames MBIA Insurance Corp. of Illinois to National Public Finance Guarantee Corporation, to focus on U.S. public finance.	Significant restrictions imposed by the New York State Insurance Department on financial guarantee insurers go into effect including limitations on the sale of guarantees and insurance on CDS. Majority of bond insurers rated below investment grade (less than BBB).

[29] NYSID means "the New York State Insurance Department".

In 1995 the New York State Insurance Department (NYSID) enabled the bond insurers to write insurance on GICs. Here is a description on a GIC mentioned in Drake and Neale (2010). Not all the funds (obtained through a municipal bond issuance) are needed at once. The temporary spare part could be deposited in a GIC (most probably operated by a bond insurer's investment management subsidiary), which promises a higher rate of return than in a money market account. The bond insurer generally invests the funds and makes profit on the difference between what it actually earns and the promised return paid out. For this product, it is crucial for bond insurers to maintain a good rating. Once a bond insurer with GIC is downgraded, the bond insurer may be required to post collateral for the GIC or pay to terminate the agreement, resulting in a significant loss of liquidity for the bond insurer.

Table III.2 displays MBIA corp.'s net par amount written by different products at the end of 2006, 2007 and 2008. We may find that the insurance products on structured finance and credit default swaps peak as of the end of 2007.

Table III.2: MBIA Corp. --Net Par Amount Written by Products (in billions)

	12/31/2006	12/31/2007	12/31/2008
US Public Finance	$46.4	$54.1	$195.1
Non-US Public Finance	$6.9	$5.5	$1.5
US Structured Finance	$39.8	$73.8	$6.9
Non-US Structured Finance	$19.1	$14.6	$3.4
Credit Default Swaps (notional value)	$139.7	$200.4	$167.9

Data source: MBIA's annual reports
Note: In its annual reports, there is no detailed amount on GIC.

Bond insurers not only provide insurance for public and structured finance, but also invest in public and structured finance insured by the same or other bond insurers. For example, in MBIA's 2008 annual report, it is stated that

MBIA.'s "investment portfolio includes investments that are insured by MBIA Corp.("MBIA Insured Investments"). As of December 31, 2008, MBIA Insured Investments at fair value represented $3.5 billion or 17% of the total investment portfolio."

Table III.3 exhibits MBIA's self-insured investment at fair value and its percentage in total investment at the end of 2006, 2007 and 2008. MBIA kept a relatively constant percentage of self-insured investment in its total investment.

Bond insurers also invest in the bonds insured by other bond insurers. We cannot get the exact proportion for that part of investment, based on their annual report.

Table III.3: MBIA Corp. --Self-insured Investment at fair value

in billions	2006	2007	2008
Self-insured Investment	$6.8	$6.8	$3.5
Percentage in Total Investment	18%	16%	17%

Data source: MBIA's annual reports

Before 2007 few bond insurers had been downgraded. Almost all of them had the best credit ratings. They usually ran their business largely unnoticed. In 2007, due to downgrading originally high quality subprime structured securities, the deteriorating condition of bond insurers started to attract attention. Table III.4 shows the major bond insurers' ratings[30] as of February 24, 2010, compared with the ones as of Jun 5, 2007. Their credit ratings have been deteriorated rapidly.

[30] Before 1990, there were 'big four' bond insurers (AMBAC, MBIA Insurance Corporation, Financial Guaranty Insurance Company and FSA). In the late 90s/2000s a few more came up, for example ACA Financial Guaranty Corp, XL Capital, and CIFG.

Table III.4: Bond Insurer Ratings
as of February 24, 2010
(initial ratings/outlooks as on 6/5/07)

Bond Insurer (Stock Ticker)	Moody's		Standard & Poor's		Fitch	
	Date	Rating	Date	Rating	Date	Rating
ACA Financial Guaranty Corp. (ACA)		NR	6/5/07 12/15/08	A- W		NR
Ambac (Ambac Assurance Corporation) (ABK)	6/5/07 7/29/09	Aaa Caa2	6/5/07 11/18/09	AAA CC+	6/5/07 6/26/08	AAA W
CIFG	6/5/07 11/11/09	Aaa W	6/5/07 2/16/10	AAA W	6/5/07 10/21/08	AAA W
FGIC (Financial Guaranty Insurance Co.)	6/5/07 4/14/09	Aaa W	6/5/07 4/22/09	AAA W	6/5/07 11/24/08	AAA W
National Re (National Public Finance Guarantee, (formerly MBIA) (MBI)	6/5/07 6/25/09	Aaa Baa1	6/5/07 9/28/09	AAA A	6/5/07 6/26/08	AAA W
Syncora Guarantee Inc. (formerly XL Capital Assurance Inc.) (SCA)	6/5/07 3/9/09	Aaa Ca	6/5/07 4/27/09	AAA R	6/5/07 9/5/08	AAA W

Note: NR means "Not Rated"; W means "Rating Withdrawn"; R means "Regulatory Action".
Source: Stone & Youngberg LLC

The credit downgrade of bond insurers may cause a series of effects and worsen the credit crunch. Chart III.1 describes the cycle graphically.

Step 1: The downgrade of part of the insured bonds' ratings due to rising defaults may worsen the bond insurers' financial situations and cause downward rating adjustments of the bond insurers.

Cycle 1:

Step 2(1): As a result, all the insured bonds' ratings are reduced.

Step 3 (1): The losses and loss reserves are increased correspondingly. It not only happens to the insurance portfolio, but also the investment portfolio, especially the one by the (same or other) bond insurers.

Step 4 (1): It further deteriorates the bond insurers' ratings.

Cycle 2:

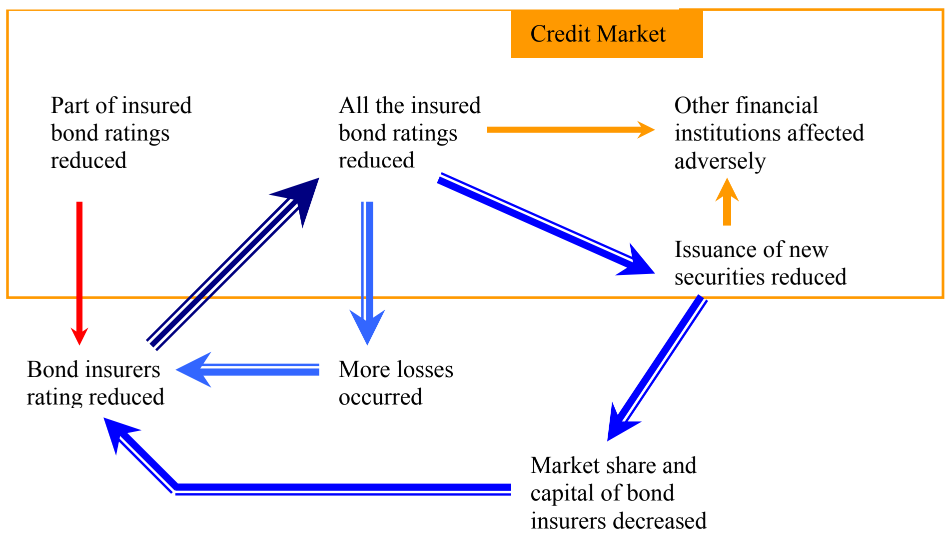
Step 2 (2): As a result, all the insured bonds' ratings are reduced.

Step 3 (2): The new issuance of public and structured finance is then decreased.

Step 4 (2): With rising claims and no new revenues, the market share and capital of bond insurers are lowered.

Step 5 (2): It further deteriorates the bond insurers' ratings.

Chart III.1: Flowchart of Bond Insurers' downgrading



Step 6: Besides, the downward ratings of all the insured bonds and less new issuance will have a knock-on effect on other financial institutions that rely on bond insurers' guarantees, which will further worsen the credit crunch.

In this chapter, we analyze the possible losses due to bonds' default and bond insurers' downgrading for currently issued bonds. Then we figure out smoothing rules for capital.

III.3. The Model

This section includes 5 parts:

1) Choose appropriate stochastic processes to capture the dynamics of bond values and the correlations between different bonds, which fluctuate through a business cycle.

2) Use the mark-to-market criteria to identify the possible loss distribution for one bond over time. The potential losses include either loss at default or loss due to bond insurers' downgrading.

3) Identify the total loss distribution for a bond insurer. Both the liability side and the asset side are considered.

4) Calculate economic capital, based on the total loss distribution.

5) Develop smoothing rules in order to dampen the procyclicality of economic capital.

6) Obtain the cumulative capital buffer.

III.3.1 Stochastic Process

Following Merton (1974)'s structural model, we assume that each bond corresponds to one project (or one firm). The funds available to the bond issuer for bond repayment by project *i* are represented by V_i, which follow the stochastic process:

$$\frac{dV_i(t)}{V_i(t)} = r(t)dt + \sigma_i(t)\left[a_i(t)dW_m(t) + \sqrt{1-a_i^2(t)}dW_i(t)\right],\ i = 1,\cdots,N. \qquad \text{(III.1)}$$

In this equation, $r(t)$ refers to the risk-free interest rate; $\sigma_i(t)$ is the instantaneous volatility of the cash flows by project *i*; $W_m(t)$ and $W_i(t)$ are two standard Wiener process, where $W_m(t)$ refers to the systemic risk / market risk factor and $W_i(t)$ is an idiosyncratic component. And we assume that

$$E[dW_m(t)dW_i(t)] = 0,$$

and $E[dW_j(t)dW_k(t)] = 0$, $\forall j,k \in \{1,...,N\}, j \neq k$.

We use $W_m(t)$ to identify the business cycles. Specifically speaking, we connect $W_m(t)$ with bond yield spread[31]. Generally, widened spreads implies that the market is factoring more risk of default, which is always related with certain degrees of credit crisis. Therefore, when bond yield spreads are high enough, e.g. $W_m(t) \geq \overline{W}$, we assume that the economy is in crisis. Otherwise, the economy is in non-crisis.

Additionally, the correlation factor $a_i(t)$ is assumed to change over the cycle. That is,

$$a_i(t) = \begin{cases} \overline{a}_i, W_m(t) \geq \overline{W} \\ \underline{a}_i, W_m(t) < \overline{W} \end{cases}. \tag{III.2}$$

And $|\overline{a}_i| > |\underline{a}_i|$, $-1 \leq \overline{a}_i < 0$, $-1 \leq \underline{a}_i \leq 1$. The assumptions mean that bonds tend to have higher correlation during financial crisis.

Therefore, the total correlation between $\frac{dV_j(t)}{V_j(t)}$ and $\frac{dV_k(t)}{V_k(t)}$ is

$$E\left[\frac{dV_j(t)}{V_j(t)}\frac{dV_k(t)}{V_k(t)}\right] = \sigma_j(t)\sigma_k(t)a_j(t)a_k(t)dt.$$

The stochastic instantaneous interest rate is assumed to follow the Cox-Ingersoll-Ross (CIR, 1985) process:

$$dr(t) = a[b - r(t)]dt + \sigma_r\sqrt{r(t)}\left[a_r(t)dW_m(t) + \sqrt{1 - a_r^2(t)}dW_r(t)\right] \tag{III.3}$$

[31] In section 4, we explain why we choose bond yield spread in detail.

where the strictly positive parameter a governs the speed of adjustment, b shows the long run value, σ_r is the volatility of interest rate changes[32], $a_r(t)$ is the correlation factor with the systemic risk, and $W_r(t)$ is a standard Wiener process.

III.3.2 Loss Distribution

For each bond, the losses faced by bond insurers can be decomposed into two parts: losses at bond's default and losses due to bond insurers' downgrading.

III.3.2.1 Loss at Default

Based on the model of Merton (1974, 1977), a bond defaults when the value of its cash flows $V_i(t)$ drops below the value of its payments at time t.

In this paper, we only analyze zero-coupon bonds, since coupon-paying bonds can be decomposed into the sum of several zero-coupon bonds.

A zero-coupon bond i may default only when the bond matures. And it happens if the bond's total asset value V_i falls below its face value. Assuming no violation of the absolute priority rule, with face value F_i and maturity T_i, the value to debt-holders at the maturity is $\min[V_i(T_i), F_i]$.

With bond insurance, the loss at default faced by bond insurer will be

$$L_i^1(T_i) = \max[0, F_i - V_i(T_i)] \qquad \text{(III.4)}$$

III.3.2.2 Loss due to Bond Insurers' Downgrading

[32] An non-positive interest rate can be precluded if a and b have non-negative values and the condition $2ab > \sigma_r^2$ is met.

Before the insured bonds default, based on mark-to-market rule, a bond insurer may still face some losses due to the market value changes caused by its downgrade. It has been observed during the latest crisis since 2007.

Most of the insured bonds have the original rating of BBB to A before insurance. So, when a bond insurer faces a downgrade, such as from AAA to below BBB, the credit insurance becomes worthless and the prices of all the bonds insured by this insurer will fall to the prices of the risky bonds with no insurance and all the other same conditions. The bond insurer's downgrade will cause market value declines of all its insured bonds.

The timing of such losses is linked to the bond insurer's downgrade, which is caused by a serious credit crisis in most cases. Therefore, the time point can be expressed as:

$$\tau_i = \{t : W_m(t) \geq \overline{W}, \& V_i(t) > F_i\}$$

Here $\overline{W}$ refers to the threshold of a credit crisis. If $W_m(t)$, represented by the bond yield spread, is no less than $\overline{W}$, a credit crisis occurs and so does the downgrading of bond insurers.

Obviously, for one specific bond, the time point of downgrading losses should be earlier than its maturity, i.e. $\tau_i < T_i$, otherwise the bond insurer needs to pay out $L_i^1(T_i)$ at the time of default.

The price of an uninsured zero-coupon bond at time *t* with risk-neutral probability *Q* is:

$$P_i^N(t) = E_t^Q\left[e^{-\int_t^{T_i} r(s)ds} \min\{F_i, V_i^Q(T_i)\}\right].$$

On the contrary, the price at time *t* of an insured zero-coupon bond is

$$P_i^I(t) = E_t^Q\left[e^{-\int_t^{T_i} r(s)ds} F_i\right].$$

In this equation, we do not consider the default possibility of the bond insurer, since the purpose of this paper is to help bond insurers keep enough capital to avoid the downgrading and default risk.

Therefore the loss for bond i at time τ_i due to the bond insurer's downgrade is the value of credit insurance:

$$L_i^2(\tau_i) = P_i^I(\tau_i) - P_i^N(\tau_i) \tag{III.5}$$

In total, the loss for one insured bond is

$$L_i(t) = \begin{cases} L_i^1(T_i), & if \quad t = T_i \\ L_i^2(\tau_i), & if \quad t = \tau_i \end{cases}, \tag{III.6}$$

depending on the different occurring time. According to our assumptions, the two parts of loss $L_i^1(T_i)$ and $L_i^2(\tau_i)$ can not occur at the same time. We only count one part of them at some time point when calculating the loss for one insured bond.

III.3.3 Total Losses for a Bond Insurer

As we mentioned in the previous part, bond insurers not only provide insurance for public and structured finance, but also invest in public and structured finance insured by other bond insurers. So we need to consider both sides of the financial statement: liability and investment.

On the liability side, we assume a bond insurer provide insurance for n_1 insurance contracts. The losses should be

$$L(t) = \sum_{i=1}^{n_1} L_i(t). \tag{III.7}$$

On the asset side, if certain investment portfolio is insured by some other bond insurer, the bond insurer will only face the losses due to bond insurers' downgrading, assuming no default risk of bond insurers. Suppose a bond insurer invests in n_2 insured bonds. Then the losses should be

$$L^A(t) = \sum_{j=1}^{n_2} L_j^A(t), \tag{III.8}$$

where $L_j^A(t) = L_j(t)$ and $t \neq T_i$, assuming no default risk for the other bond insurers.

Here we assume the bond insurers are perfectly correlated. That is to say, when one bond insurer is downgraded, the rest bond insurers will be downgraded too. This assumption is consistent with most bond insurers downgrading since 2007.

Combining both sides, the total losses for a bond insurer are

$$L^T(t) = L(t) + L^A(t). \tag{III.9}$$

III.3.4 Economic Capital

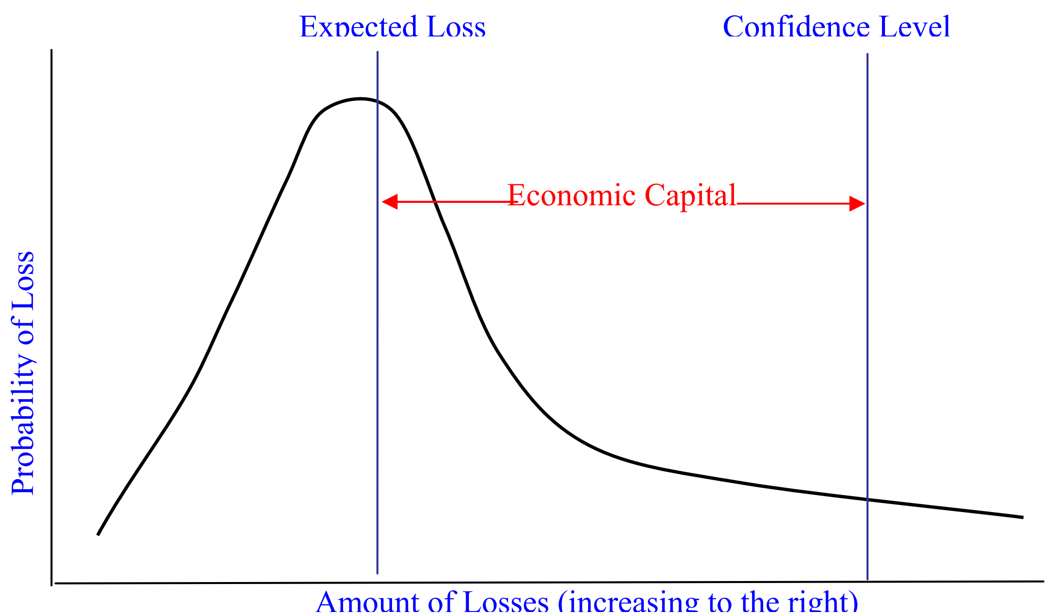
According to SOA's Special Guide on Economic capital (2004), Economic Capital[33] can be defined as sufficient surplus to cover potential future losses, at a given risk tolerance level. Therefore, economic capital is calculated based on different risks assumed and is a forward-looking measure of capital adequacy. Corresponding to the

[33] Since regulatory capital or rating agency capital is closely related with economic capital and can be somehow converted to an Economic Capital equivalent, we only investigate economic capital in this paper. Regulatory capital is minimum capital required by the corresponding regulatory agencies. Rating agency capital is the capital calculated by rating agencies, such as Moody's, Standard &Poor's and A.M. Best, to determine the company financial strength ratings.

assumed risks, we use market value (instead of book value) of assets and liabilities to estimate economic capital.

In practice, economic capital is estimated against unexpected future losses at a selected confidence level (Chart III.2).

Chart III.2: Economic Capital



Generally, economic capital can cover all kinds of risks a company is facing. In this paper, we mainly consider the default and market risks contained in the insurance products and investment portfolios mentioned in the previous section. That is, the risks here refer to $L^T(t)$. $F_{L^T}(x)$ refers to the corresponding cumulative probability distribution function.

And we utilize the most commonly used methods, Value at Risk (VaR), to calculate economic capital. Suppose α is the given percentile of the loss distribution. Then the VaR at the end of each period (i.e. quarter) for the total loss at that period is

$$VaR_\alpha\left(L^T\right)=\min\left\{x\in R:F_{L^T}(x)\geq\alpha\right\}=F_{L^T}^{-1}(\alpha) \qquad \text{(III.10)}$$

The economic capital, EC, at the beginning of each period is the VaR minus the expect value of the total loss for the same period, discounted backward one period at the risk free rate. Here we assume the capital is invested in risk-free assets for the single period.

III.3.5 Capital Smoothing

Short-term-oriented economic capital basically provides protection against potential losses for that certain period. Under this situation, economic capital may show pro-cyclical trend over business cycles:

1) During booms, the potential losses could be underestimated. So is the economic capital;
2) In the crisis, the potential losses should be much higher. Correspondingly, the company has to prepare a large amount of economic capital, which worsens its financial situations further.

The potential procyclicality of the New Basel Capital Accord in the bank industry has been extensively discussed. The possible solutions include:

1) Kashyap and Stein (2004) point out that the target solvency probability in the regulatory rule should be time-varying over the cycle, because the shadow cost of bank capital varies. This argument can also be used in the internal system.

2) Gordy and Howells (2006) explore several smoothing rules for capital. For example:

a) Autoregressive Filter

$$\hat{C}_{i,t} = \widehat{C}_{i,t-1} + \alpha\left(C_{i,t} - \hat{C}_{i,t-1}\right)$$

where $C_{i,t}$ is the unsmoothed output from the capital formula for bank *i* at time *t*; $\widehat{C}_{i,t}$ denotes the smoothed capital requirement; and α is an adjustment parameter controlling the degree of smoothing.

b) Counter-cyclical Indexing

$$\hat{C}_{i,t} = \alpha_t C_{i,t}$$

where the smoothing parameter α_t changes correspondingly in each period, with the mean equal to one. And α_t should be greater than one in good economy and less than one in crisis.

Additionally, Balla and McKenna (2009) shows the Spanish model of dynamic provisioning as a countercyclical tool for loan loss reserves.

$$General \quad provision_t = \sum_{i=1}^{6} \alpha_i \Delta A_{it} + \sum_{i=1}^{6} \left(\beta_i - \frac{Special \quad provision_{it}}{A_{it}} \right) A_{it}$$

In this model, there exist six risk categories with different α or β assigned. A_{it} denotes the credit volume of asset *i* at time *t*.

All the above literature states the smoothing methods using the historical or current data. In this paper, we apply a forward-looking procedure to economic capital calculations in order to dampen procyclicality.

$$E\hat{C}_t = EC_t + \omega_t U_t \qquad \text{(III.11)}$$

where EC_t denotes the economic capital at time *t* calculated by VaR in this paper. $E\widehat{C}_t$ refers to the smoothed economic capital at time *t*.

Most importantly, U_t is determined over one full business cycle. For example, it can be

$$U_t = \overline{EC}_t - EC_t \qquad \textit{(option 1)}$$

$$= \frac{1}{T-t+1}\sum_{i=t}^{T} e^{-\int_t^i \mu(s)ds} EC_i - EC_t \,. \qquad \text{(III.12)}$$

$\overline{EC}_t$ refers to the average of predicted economic capital over the full business cycle, discounted to time *t* at the cost of capital.

$\omega_t \geq 0$ is the adjustment parameter. If $\omega_t = 0$, there is no adjustment and $E\hat{C}_t = EC_t$. Otherwise $\omega_t > 0$. In this case,

$$\begin{cases} E\widehat{C}_t < EC_t, & if \quad EC_t > \overline{EC} \\ E\widehat{C}_t > EC_t, & if \quad EC_t < \overline{EC} \end{cases}$$

Some alternative definition can be used for U_t.

$$U_t = \overline{EC}_t + \eta \cdot \sigma\left(EC_{t,t+\Delta}\right) - EC_t \qquad \textit{(option 2)} \qquad \text{(III.13)}$$

where $\eta > 0$ is an adjustment parameter; $\sigma\left(EC_{t,t+\Delta}\right)$ is the standard deviation of predicted discounted economic capital over the full business cycle. That is

$$EC_{d,t_2} = e^{-\int_t^{t_2} \mu(s)ds} EC_{t_2}$$

(Option 2) is more conservative than (Option 1) of U_t. Only when $EC_t > \overline{EC} + \eta \cdot \sigma\left(EC_d\right)$, instead of $EC_t > \overline{EC}$, will $E\widehat{C}_t$ be adjusted downward, compared to EC_t. With all the other conditions same, (Option 2) is just a parallel shift of (Option 1).

The third alternative for U_t can be related with the total amount guaranteed.

$$U_t = \overline{EC}_t + \tau_t \Delta P_t - EC_t \qquad \textit{(option 3)} \qquad \text{(III.14)}$$

where $\tau_t > 0$ is the adjustment parameter. $\Delta P_t = P_t - P_{t-1}$ refers to the change of total contract amount guaranteed by the bond insurer from time *t-1* to time *t*. Besides the insurance contract, P_t should also include the investment insured by the same or other bond insurers. So, $\Delta P_t > 0$ means the business of the bond insurer is in expansion, and U_t can be larger. On the contrary, $\Delta P_t < 0$ means that its business is in bad time, maybe in crisis, and then U_t can be smaller.

The features of smoothing methods mentioned here are:

1) The smoothing is forward-looking. It utilizes the future estimated economic capital to adjust today's value.
2) The adjustment is based on a full business cycle, rather than one or two periods, to avoid underestimating the potential losses.
3) When the adjustment parameter ω_t is greater than one, the adjusted capital can be counter-cyclical.

III.3.6 Establishing Capital Buffer

With the smoothed capital, a bond insurer can actually establish some capital buffer in good times to support the potential losses in crisis.

Suppose the capital buffer established since time *1*. Then the cumulative capital buffer at time *t* would be:

$$CB_t = \sum_{j=1}^{t} \left(E\hat{C}_j - EC_j \right) = \sum_{j=1}^{t} \omega_j U_j$$

$$= \sum_{j=1}^{t} \omega_j \left(\overline{EC}_j - EC_j \right) \qquad \textit{(option 1)}$$

$$\text{or } = \sum_{j=1}^{t} \omega_j \left[\overline{EC}_j + \eta \cdot \sigma\left(EC_{j,j+\Delta} \right) - EC_j \right] \qquad \textit{(option 2)}$$

$$\text{or } = \sum_{j=1}^{t} \omega_j \left[\overline{EC}_j + \tau_j \Delta P_j - EC_j \right] \qquad \textit{(option 3)}$$

III.4. Modeling the Business Cycle

The United States National Bureau of Economic Research (NBER) identifies the dates of the peaks and troughs of the business cycle in the USA[34]. Then an expansion is the period from a trough to a peak, and a recession is the period from a peak to a trough. Table III.5 shows the data released on the NBER website.

[34] The determinants of the business cycle phases include real GDP, real income, employment, industrial production, and wholesale-retail sales.

Table III.5: US Business Cycle Expansions and Contractions by NBER

BUSINESS CYCLE REFERENCE DATES		DURATION IN MONTHS			
Peak	**Trough**	**Contraction**	**Expansion**	**Cycle**	
Quarterly dates are in parentheses		*Peak to Trough*	*Previous trough to this peak*	*Trough from Previous Trough*	*Peak from Previous Peak*
	December 1854 (IV)	--	--	--	--
June 1857(II)	December 1858 (IV)	18	30	48	--
October 1860(III)	June 1861 (III)	8	22	30	40
April 1865(I)	December 1867 (I)	32	46	78	54
June 1869(II)	December 1870 (IV)	18	18	36	50
October 1873(III)	March 1879 (I)	65	34	99	52
March 1882(I)	May 1885 (II)	38	36	74	101
March 1887(II)	April 1888 (I)	13	22	35	60
July 1890(III)	May 1891 (II)	10	27	37	40
January 1893(I)	June 1894 (II)	17	20	37	30
December 1895(IV)	June 1897 (II)	18	18	36	35
June 1899(III)	December 1900 (IV)	18	24	42	42
September 1902(IV)	August 1904 (III)	23	21	44	39
May 1907(II)	June 1908 (II)	13	33	46	56
January 1910(I)	January 1912 (IV)	24	19	43	32
January 1913(I)	December 1914 (IV)	23	12	35	36
August 1918(III)	March 1919 (I)	7	44	51	67
January 1920(I)	July 1921 (III)	18	10	28	17
May 1923(II)	July 1924 (III)	14	22	36	40
October 1926(III)	November 1927 (IV)	13	27	40	41
August 1929(III)	March 1933 (I)	43	21	64	34
May 1937(II)	June 1938 (II)	13	50	63	93
February 1945(I)	October 1945 (IV)	8	80	88	93
November 1948(IV)	October 1949 (IV)	11	37	48	45
July 1953(II)	May 1954 (II)	10	45	55	56
August 1957(III)	April 1958 (II)	8	39	47	49
April 1960(II)	February 1961 (I)	10	24	34	32
December 1969(IV)	November 1970 (IV)	11	106	117	116
November 1973(IV)	March 1975 (I)	16	36	52	47
January 1980(I)	July 1980 (III)	6	58	64	74
July 1981(III)	November 1982 (IV)	16	12	28	18
July 1990(III)	March 1991(I)	8	92	100	108
March 2001(I)	November 2001 (IV)	8	120	128	128
December 2007 (IV)			73		81
Average, all cycles:					
1854-2001 (32 cycles)		17	38	55	56
1854-1919 (16 cycles)		22	27	48	49
1919-1945 (6 cycles)		18	35	53	53
1945-2001 (10 cycles)		10	57	67	67

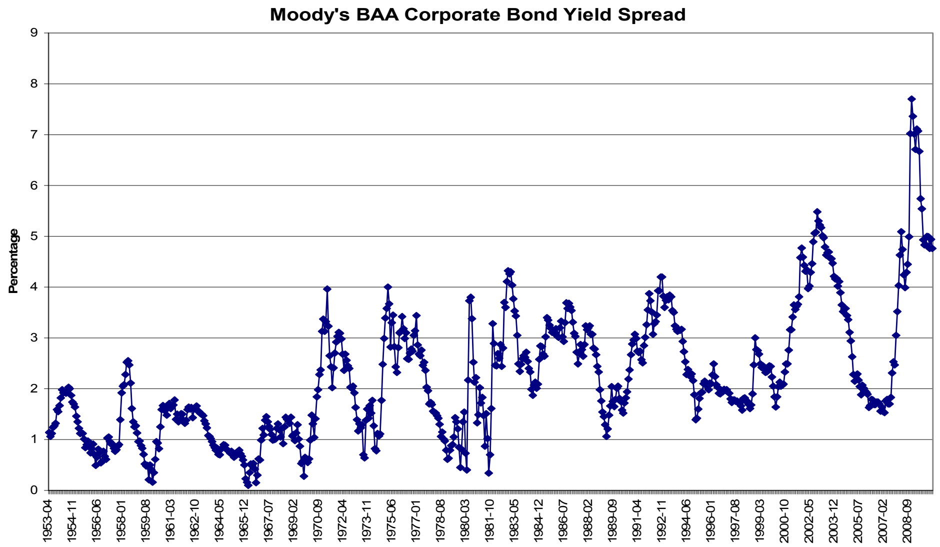
In our model, the economy is either in crisis state or in non-crisis state, instead of expansion or recession in the NBER business cycle. The identification is due to the features of bond insurers.

1) Bond insurers' operations heavily depend on credit market. Although credit market is closely related with the macro-economic variables, there may exist several-period lags. Additionally, not all the recessions will cause serious problems in credit market.
2) Only crisis or deep recession may trigger the downgrading of bond insurers. A mild and short economic downturn only moderately increases the losses suffered by bond insurers, while severe downturns may cause catastrophic losses, which could downgrade bond insurers.

So we choose a bond yield spread index as the variable representing the cycles. Specifically, we use the balance of Moody's BAA corporate bond yield and Treasury bond rate to identify the cycles. We also checked Moody's AAA corporate bond yield index and municipal bond indices. The identifications of cycles are basically the same. Since Moody's corporate bond yield index provides more available data and most insured bonds have the rating around BAA, we choose Moody's BAA corporate bond yield index (Chart III.3).

Chart III.3: Moody's BAA Corporate Bond Yield Spread

This chart shows monthly data of Moody's BAA Corporate Bond Yield Spread from April 1953 till January 2010.

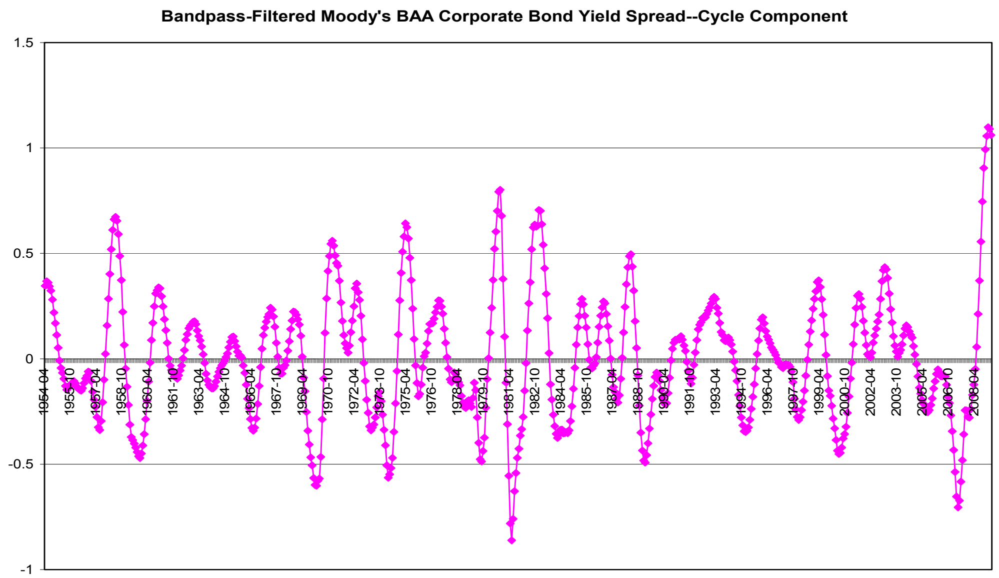


The corporate bond yield index itself contains long-run trend and some short-term noises. So we adopt the approximate band-pass filters proposed by Baxer and King (1999) to measure business cycles. Their procedures isolate business cycle components by applying particular moving averages and eliminate the low frequency fluctuations associated with trend growth and the high frequency fluctuations associated with short-term noises.

Chart III.4 displays the cycle component of the Bandpass-Filtered Moody's BAA Corporate Bond Yield Spread. Generally, higher bond yield spread refers to tighter credit market and even credit crisis.

Chart III.4: Bandpass-Filtered Moody's BAA Corporate Bond Yield Spread- Cycle Component

This Chart displays the cycle component of the Bandpass-Filtered Moody's BAA Corporate Bond Yield Spread. Generally, higher bond yield spread refers to tighter credit market and even credit crisis.



According to Chart III.4, we choose that the cycle component of yield spread should be greater than 0.4355 when the economy is in crisis. 0.4355 is about the highest cycle component part value of yield spread from 2003 to 2004. Obviously, there is no recession during that period, based on the NBER cycle identification. The reason why we got a relatively high value may be the low Treasury bond rates during that period.

This criterion captures the major economic troughs since 1954, compared with the NBER cycle. The differences are that 1) the length of crises in our identification is shorter than the contraction periods, and 2) there may be some lags between the troughs of the two identification methods.

To be consistent with the definitions in the model part, we normalize the criterion by adjusting the mean and standard deviation. That is

$$\overline{W} = \frac{0.4355 - 0.003065}{0.2934} = 1.4738$$

This is the unconditional threshold for the systemic risk factor.

According to the unconditional threshold, 46 out of 660 months between April 1954 and March 2009 were classified as "crisis" and the remaining 614 as "non-crisis". Therefore, if we simply select a month at random, there is a 7% probability that it is classified as a crisis and a 93% probability of falling in a non-crisis status. These are the unconditional probabilities. Let $S_t = 2$ if a month t is regarded as a crisis, otherwise $S_t = 1$. Then

$$\begin{cases} \Pr(S_t = 1) = 93\% \\ \Pr(S_t = 2) = 7\% \end{cases}$$

Additionally, of 46 months characterized as crisis, 39 or 85% were followed by another month of crisis. Of 614 non-crisis months, 607 or 99% were followed by another non-crisis month. Therefore the conditional probabilities are

$$\begin{cases} \Pr(S_{t+1} = 1 | S_t = 1) = 99\% \\ \Pr(S_{t+1} = 2 | S_t = 1) = 1\% \\ \Pr(S_{t+1} = 1 | S_t = 2) = 15\% \\ \Pr(S_{t+1} = 2 | S_t = 2) = 85\% \end{cases}$$

The Markov transition matrix is $\begin{pmatrix} 99\% & 1\% \\ 15\% & 85\% \end{pmatrix}$.

Based on the conditional probabilities, the conditional thresholds for $\overline{W}$ should be adjusted correspondingly.

If $S_t = 1$, then at time $t+1$, the conditional threshold $\overline{W} = \overline{W}_1 = 2.3263 = \Phi^{-1}(0.99)$, where Φ refers to the cumulative standard normal distribution. That is to say, given $S_t = 1$, $S_{t+1} = 1$ if $W < \overline{W}_1 = 2.3263$ and $S_{t+1} = 2$ if $W \geq \overline{W}_1 = 2.3263$.

Accordingly, if $S_t = 2$, then at time $t+1$, the conditional threshold $\overline{W} = \overline{W}_2 = -1.0364 = \Phi^{-1}(0.15)$. So, given $S_t = 2$, $S_{t+1} = 1$ if $W < \overline{W}_2 = -1.0364$ and $S_{t+1} = 2$ if $W \geq \overline{W}_2 = -1.0364$.

Between April 1954 and March 2009, the average length in non-crisis status is 88 months and the one in crisis is 6.5-8 months. So the average length of one cycle is around 8 years. In the next simulations, we assume 8 years as the length of a full business cycle.

III.5 Simulation Results

Monte Carlo simulations are used to estimate the previous model.

III.5.1 Parameter Values

In our example, we assume that this bond insurer provide insurance for 5 bond contracts and invest in one bond portfolio insured by some other bond insurers. The parameter values for all the project cash flows processes are listed in Table III.6. All the bonds are assumed to be zero-coupon bonds with different maturities.

We assume that the bond insurer does not manage its portfolio actively. If a bond matures or defaults, the bond insurer will provide credit insurance for a new bond with the same characteristics. Thus the asset and liability sizes will remain constant over time.

Table III.6: Parameter values for bonds and corresponding project values process

This table provides the parameter values for 5 insured bonds (and corresponding projects) and one investment insured by the same bond insurer.

$$\frac{dV_i(t)}{V_i(t)} = r(t)dt + \sigma_i(t)\left[a_i(t)dW_m(t) + \sqrt{1-a_i^2(t)}dW_i(t)\right]$$

i	Initial value of project	Face Value	Correlation factor in crisis status	Correlation factor in non-crisis status	Volatility	Maturity of bond
	$V_i(0)$	F_i	$\overline{a}_i$	$\underline{a}_i$	σ_i	T_i
1	10000	6000	-0.8	-0.4	0.15	8
2	10000	6000	-0.8	-0.4	0.3	8
3	10000	7000	-0.8	-0.6	0.15	8
4	10000	8000	-0.8	-0.5	0.15	4
5	10000	8000	-0.8	-0.4	0.3	4
A	10000	6000	-0.8	-0.4	0.15	8

The parameter values of interest rate process (equation 3) are listed in Table III.7. The initial interest rate ($r(0)$) is assumed to be 5 percent; the long run value (*b*) is 5 percent; the speed of adjustment (*a*) is 0.2; the volatility of interest rate changes (σ_r) is 0.05; the correlation factor (a_r) is -0.5, since the systemic risk factor is represented by yield spread.

Table III.7: Parameter values for interest rate process

This table shows the parameter value of interest rate process following the equation

$$dr(t) = a[b - r(t)]dt + \sigma_r\sqrt{r(t)}\left[a_r(t)dW_m(t) + \sqrt{1-a_r^2(t)}dW_r(t)\right]$$

Parameter	$r(0)$	a	b	σ_r	a_r
Value	5%	0.2	5%	0.05	-0.5

III.5.2 Simulation Steps

Using these parameters, we run simulations to get the insurer's total loss distribution. We generate 200 economic scenarios and 100 simulations of final project values in each scenario, with the total of 20000 paths. The detailed simulation steps are as follows:

1) Use the unconditional probability $\begin{bmatrix}93\% & 7\%\end{bmatrix}$[35] to simulate whether the economy is in crisis or non-crisis at the beginning of the period (t=0).
2) Simulate 200 paths of economic scenarios for 8 years, based on the conditional probabilities. Along each path, we got the periods in crisis.
3) Simulate systemic risk factor. If the economy in the previous period is in non-crisis and in the current period is in non-crisis, the systemic risk factor for the current period is chosen below 2.3263. If the economy in the previous period is in non-crisis and in the current period is in crisis, the systemic risk factor is chosen above 2.3263. Similarly, if the economy in the previous period is in crisis and in the current period is in non-crisis, the systemic risk factor is chosen below -1.0364. If the economy in the previous period is in crisis and in the current period is in crisis, the systemic risk factor is chosen above -1.0364.
4) Simulate interest rate for each economic scenario.
5) Generate 100 project values for each contract under each economic scenario.
6) Calculate the loss at default for each contract (at the maturity).
7) Calculate the losses in crisis[36], based on the economic status. From step 6) and 7), we obtain the total loss distributions for each period along a whole business cycle horizon.

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[35] The unconditional probabilities are consistent with the unconditional threshold for the systemic risk factor.

8) Estimate the value of economic capital at the beginning point of one period for each period, using the methodology described in section III.3.4.

III.5.3 Calculating Unsmoothed and Smoothed Economic Capital

We choose the parameter α for VaR as 99% to calculate the unsmoothed economic capital. The economic capitals over a business cycle, discounted to the value at time 0, are shown in Chart III.5 and III.6. We can observe a higher degree of capital fluctuation.

Following Equation (III.11) and (III.12), we can obtain the smoothed economic capitals by assuming ω_t. Chart III.5 displays the smoothed economic capitals when $\omega_t = 0.4,\quad 0.7,\quad 1.2$. Obviously, when $\omega_t < 1$, higher ω_t could make the adjusted economic capital smoother over time. When $\omega_t = 1$, the smoothed economic capital will just equal to the averaged economic capital. If $\omega_t > 1$, the adjusted economic capital will be totally counter-cyclical.

Following the rules shown in Equation (III.11) and (III.13), we need assume both ω_t and η to obtain the smoothed economic capitals. Chart III.6 displays the smoothed economic capitals when $\omega_t = 0.7$ and $\eta = 0,\quad 0.1,\quad 0.3$. With η changing, the adjusted economic capital move parallel.

As for the smoothing rule in Equation (III.11) and (III.14), ΔP_t is not based on simulations and should be the change of real guaranteed amounts. We will not show the results here.

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[36] We assume the bond insurers are downgraded in crisis.

Chart III.5: Unsmoothed vs Smoothed (Option 1) Economic Capital

This chart shows the unsmoothed economic capital and smoothed economic capital using option 1 (equation III.11 and III.12). The smoothed economic capitals when $\omega_t = 0.4,\ \ 0.7,\ \ 1.2$ are displayed. When $\omega_t < 1$, higher ω_t could make the adjusted economic capital smoother over time. If $\omega_t > 1$, the adjusted economic capital will be totally counter-cyclical.

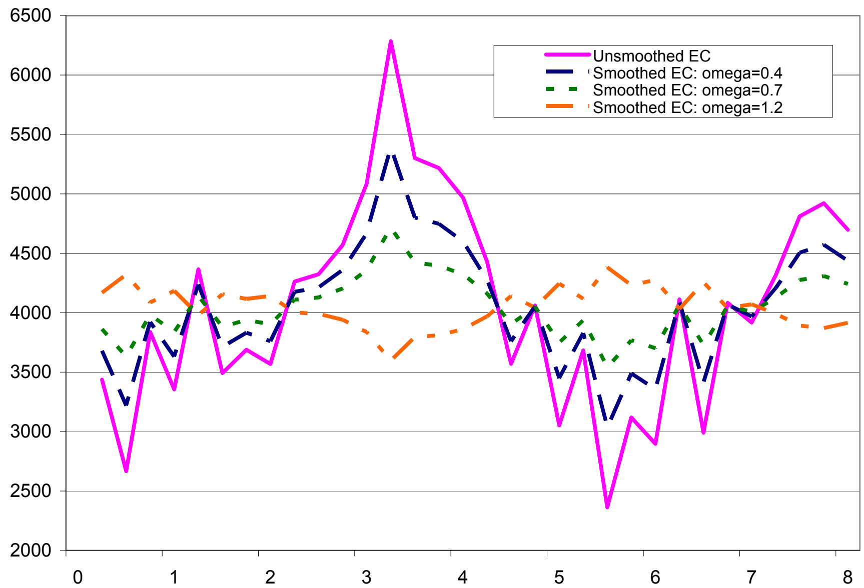
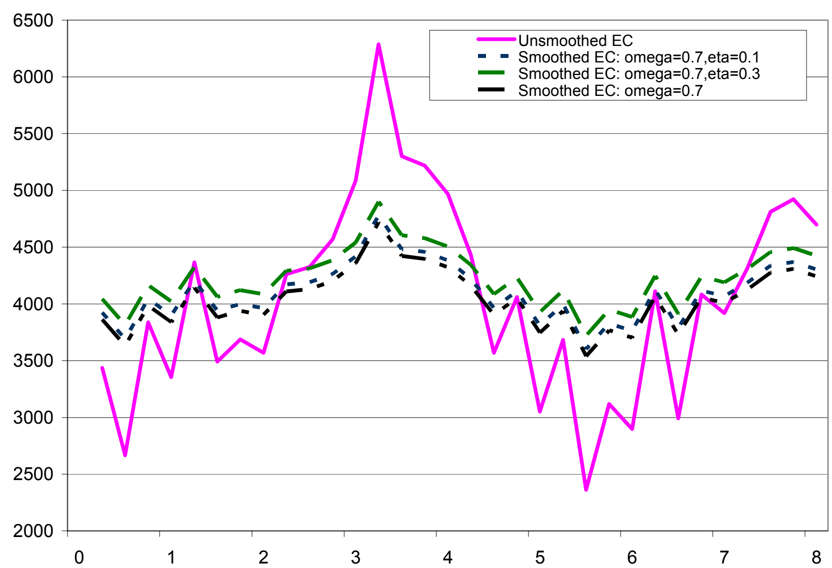


Chart III.6: Unsmoothed vs Smoothed (Option 2) Economic Capital

This chart shows the unsmoothed economic capital and smoothed economic capital using option 2 (equation III.11 and III.13). The smoothed economic capitals when $\omega_t = 0.7$ and $\eta = 0,\ \ 0.1,\ \ 0.3$ are displayed. With η changing, the adjusted economic capital move parallel.



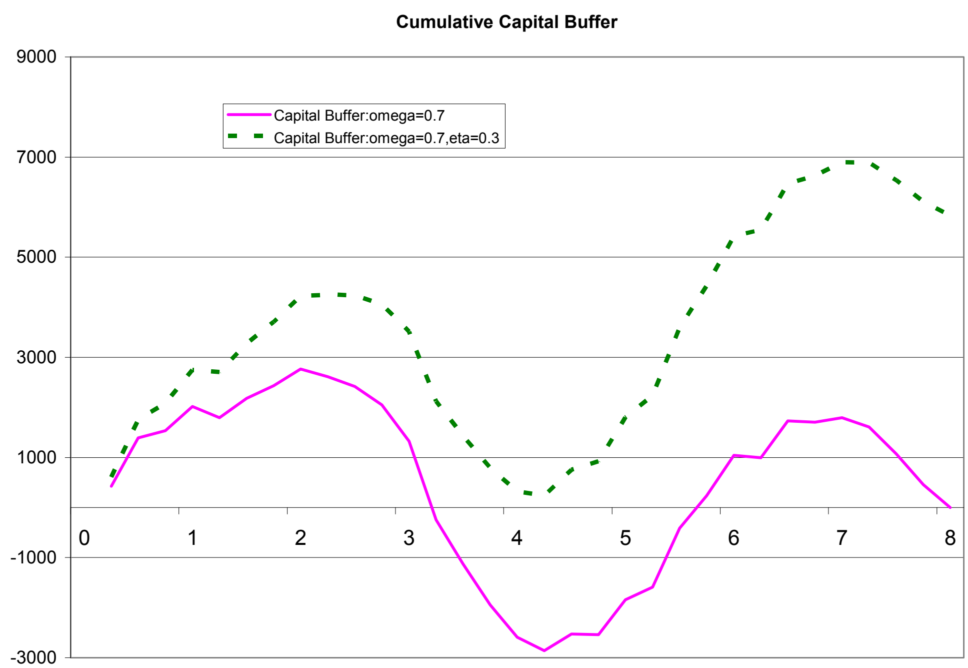
III.5.4 Capital Buffer

Based on the differences between smoothed and unsmoothed economic capital, we may establish some appropriate capital buffer, so that the extra capital in good time could be accumulated to pay the potential losses in crisis.

Chart III.7 displays the cumulative capital buffer based on this simulated business cycle with $\omega_t = 0.7$ and $\eta = 0, \quad 0.3$. When $\eta > 0$, more capital buffers can be accumulated.

Chart III.7: Cumulative Capital Buffer

This chart displays the cumulative capital buffer based on this simulated business cycle with $\omega_t = 0.7$ and $\eta = 0, \quad 0.3$. When $\eta > 0$, more capital buffers can be accumulated.



Of course, in practice, the business cycle will be simulated at each time point. Therefore, the unsmoothed and smoothed economic capital and the cumulative capital buffer will be adjusted correspondingly.

III.6. Contingent Capital Buffer

Establishing a higher capital buffer during normal times may be costly. In order to reduce the cost of extra capital, some form of contingent capital can be applied.

For example, Flannery (2009) advocate “contingent capital certificates”, designed for individual financial institutions. The certificates are supposed to convert from debt to equity automatically when the issuer’s equity ratio falls below certain level.

Kashyap *et al* (2008) recommend “disaster insurance” for banks. Banks may acquire an insurance policy, which pays off certain pre-specified amount of money to the banks upon the occurrence of a systemic “event”. The trigger event is defined based on the aggregate performance of major financial institutions, instead of an individual institution. From the perspective of insurance provider, the insurance policy “would resemble an investment in a defaultable ‘catastrophe’ bond”.

Bond insurance industry may also design similar mechanisms either for individual bond insurer or for the whole industry. And a bond insurer could determine the amount of their contingent capital buffer, based on the calculated cumulative capital buffer in the previous section.

III.7. Conclusion

In this chapter, we describe the business features of bond insurers and the impacts of business cycles on them. Then we adopt a structural model with time-varying correlations, which are closely tied up with the business cycle. By including both losses due to bond insurers’ downgrading and losses from insurance contracts and investment portfolio, we obtain the total loss distribution and corresponding economic capital of a bond insurer. On that basis, we propose forward-looking smoothing rules of capital over a full business cycle, instead of only based on a short-term horizon, to avoid the procyclicality. The simulation results show the smoothed capital may vary from lower degree of procyclicality to totally counter-cyclicality, corresponding to the different parameter values.

Based on the smoothed economic capital, the next important step is to establish some appropriate capital buffer, so that the extra capital in good time could be accumulated to pay the potential losses in crisis. In order to reduce the cost of extra capital, some form of contingent capital can be applied.

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